



2D-simulation of the system: laser beam + laser plasma + target

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Abstract

The simulation is based on a set of two-dimensional transient equations of gas dynamics with two-dimensional radiation transfer equation completed by heat transfer equation for condensed phase. The influence of the laser radiation wavelength and the choice of the diameter of the initial high-temperature region in the gas phase (i.e. the region of optical breakdown) on the laser plasma dynamics and the transfer of the laser and plasma radiation energy to the target are analyzed.

1. Introduction

Dynamics of plasma generated by the action of pulsed laser radiation on a non-ablating surface depends on the conditions in the gas medium surrounding the target and action parameters: energy per pulse, pulse duration, wavelength, and the spatial-temporal energy distribution in the pulse. When a limited area of the target surface is irradiated by a laser beam it becomes necessary to take into account and examine the two-dimensional effects of laser heating and gas-dynamic expansion of plasma.

The temperature dynamics of the irradiated target depends on the nature of the interactions between gas dynamic processes and radiative heat-transfer in plasma. In the general case this interaction is highly non-linear and complicated. The laser wavelength is one of the main governing parameters that determine

the conditions for volume energy release in laser produced plasma.

Under similar initial conditions the optical breakdown plasma may be either completely transparent or strongly absorbing with respect to laser radiation, depending on its wavelength. The target surface may be either destroyed by laser radiation or completely screened from its action. Besides, it is known [1,2] that when plasma is formed, with gas dynamic processes being insignificant, the efficiency of energy coupling of CO₂-lasers may be enhanced by converting laser radiation into UV-radiation. In this connection it is of particular interest to analyze the dynamics of energy input into the target with respect to the laser wavelength and the initial parameters of plasma.

In the present paper mathematical modelling is used to study the influence of the laser radiation wavelength and the choice of the initial high-temperature region (i.e. the region of optical breakdown) on the laser plasma dynamics and the transfer of the laser and plasma radiation energy to the target.

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2. Formulation of the problem

One of the specific features of laser plasma development is that the initial conditions (i.e., optical break-down) affect the solution. In a cold gas the focused laser radiation is propagated without any noticeable intensity loss. If the gas is heated up to the temperature required for break-down, the gas medium becomes optically opaque and volume release of the laser energy takes place. The emergence of plasma near a target is connected with the kinetics of surface evaporation and optical break-down of gaseous media. To determine the spatial position of break-down and the threshold intensity leading to a steady absorption of radiation, entails a number of difficulties. Such problems can be solved correctly only using kinetics models [3]. To simplify the problems describing the gas dynamic stage of the laser plasma development, the initial stable absorption of radiation is usually modelled by inserting a high temperature region close to the irradiated surface [4–6].

In a general case the geometrical size of the high temperature region is connected with the action parameters: the radiation wavelength and the focusing spot radius. The region thickness ΔZ depends on the wavelength and is not known beforehand. Usually its value varies from a few tens to several hundreds of μm . The heated region radius Δr is usually associated with the focusing radius R_θ .

When the self-radiation of laser plasma is taken into account, the corresponding mathematical model becomes rather complicated. In general, the description of radiation transfer depends on the emission-absorption mechanisms, as well as on the medium parameters: its characteristic size L and free-path length l_ν of a photon (which in turn depends on the density ρ and temperature T). The main mechanisms of radiation emission-absorption are the following: (a) discrete emission-absorption in electron shells of atoms and ions (bound-bound transitions); (b) radiative electron recombination and photoionization (bound-free transitions); (c) photoemission by a free electron in the Coulomb field of an ion (bremsstrahlung) and photon absorption by an electron in an ion field (free-free transitions).

The problem of radiation transfer could be simplified in the two asymptotic cases: optically thin (l_ν

$\gg L$) and optically thick plasmas ($l_\nu \ll L$). In a general case laser plasma corresponds to an intermediate position, as a result the processes at micro- and macrolevels (i.e., kinetics at the atomic scale, energy and mass transfer governing the flow structure) should be analyzed together.

Moreover, absorption of laser radiation in a plasma pattern by its electron subsystem (due to inverse bremsstrahlung mechanism) results in a substantial temperature gap between electron and heavy particles subsystems. Hence, laser plasma occurs in thermal nonequilibrium state.

To simplify the problem of laser plasma simulation, several stages could be distinguished. Kinetics of nonequilibrium low temperature optically transparent laser plasma is considered in Ref. [3] for optical breakdown simulation. The model of level-by-level kinetics taking into account line spectrum transfer in nonequilibrium plasma with arbitrary optical density is proposed in Ref. [7]: different broadening mechanisms as well as spatial variations of temperature and density are considered.

In the present model the approximation of optically thick plasma characterized by the Saha-Boltzmann charge composition is used. The corresponding spectrum of equilibrium black body radiation contains no lines because broadening is too large and the lines merge with the continuous spectrum. In fact, the conditions of local thermodynamic equilibrium in laser plasma probably should be respected at least close to the target's surface.

Assuming that laser radiation is focused into a round spot with the radius R_θ , we can formulate thermal and gas-dynamic problems in the cylindrical system of co-ordinates with the axial symmetry. Ultimately the mathematical formulation of the problem under consideration is reduced to the non-linear transient equation of heat transfer for condensed medium

$$\rho C_p(T) \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \lambda(T) \frac{\partial T}{\partial r} \right) + \frac{\partial}{\partial z} \left(\lambda(T) \frac{\partial T}{\partial z} \right)$$

$$0 < r < r_N, 0 < z < z_M, \quad (1)$$

and the complete system of two-dimensional transient equations of gas dynamics with two-dimensional radiation transfer equation for non-viscous, non-thermal conducting, radiative gaseous medium:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \rho u) + \frac{\partial}{\partial z} (\rho v) = 0, \quad (2)$$

$$\frac{\partial \rho u}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \rho u^2) + \frac{\partial}{\partial z} (\rho u v) = - \frac{\partial (p + \omega)}{\partial r}, \quad (3)$$

$$\frac{\partial \rho v}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \rho u v) + \frac{\partial}{\partial z} (\rho v^2) = - \frac{\partial (p + \omega)}{\partial z}, \quad (4)$$

$$\begin{aligned} \frac{\partial \rho \epsilon}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \rho u \epsilon) + \frac{\partial}{\partial z} (\rho v \epsilon) \\ = -p \left[\frac{1}{r} \frac{\partial r u}{\partial r} + \frac{\partial v}{\partial z} \right] - \frac{1}{r} \frac{\partial W_r}{\partial r} - \frac{\partial W_z}{\partial z} - \frac{\partial G}{\partial z}, \end{aligned} \quad (5)$$

$$\frac{1}{r} \frac{\partial (r W_r^v)}{\partial r} + \frac{\partial W_z^v}{\partial z} + c \kappa_\nu U_\nu = c \kappa_\nu U_{\nu eq}, \quad (6)$$

$$\frac{1}{3} \frac{c}{\kappa_\nu} \frac{\partial U_\nu}{\partial r} = -W_r^v, \quad \frac{1}{3} \frac{c}{\kappa_\nu} \frac{\partial U_\nu}{\partial z} = -W_z^v, \quad (7)$$

$$\begin{aligned} \frac{\partial G^+}{\partial z} - \kappa_{\nu_0} G^+ = 0, \quad \frac{\partial G^-}{\partial z} + \kappa_{\nu_0} G^- = 0, \\ G = G^+ + G^-, \end{aligned} \quad (8)$$

$$p = p(\rho_i, T), \quad \epsilon = \epsilon(\rho_i, T), \quad \kappa_\nu = \kappa_\nu(\rho_i, T, \nu),$$

where t , r , z are temporal and spatial co-ordinates; C_p , λ are the specific heat and thermal conductivity of the condensed medium; T is the temperature; ρ , p are the density and the pressure; u , v are the velocity vector components; κ_ν is the spectral absorption coefficient; κ_{ν_0} is the one for laser wavelength; U , W are the density and intensity of plasma radiation flux; ϵ is the internal energy, ω is the artificial viscosity, c is the velocity of light, G^+ (G^-) is the intensity of laser radiation in the positive (negative) direction of z -axis. G^- is due to laser radiation reflection on the target's surface. Index i is related to the various elementary species taken into account.

The system of equations is supplemented by the boundary conditions on the irradiating surface. The condense medium surface is assumed to be non-ablating and to be influenced by the laser G_{sr} and the plasma W_{sr} radiation flows:

$$-\lambda \frac{\partial T}{\partial z} = G_{sr} + W_{sr} - \epsilon \sigma T_{sr}^4, \quad z = z_0.$$

The other boundaries of the target are assumed to be adiabatic:

$$\lambda \frac{\partial T}{\partial r} = \lambda \frac{\partial T}{\partial z} = 0, \quad r = r_0, \quad z = l.$$

For gas dynamics and radiation transfer equations the following boundary conditions are used:

$$z = z_0: u = 0, W_{sr} = W_z = cU/2, G_{sr} = A(T_{sr})G_{z_0}^+,$$

$$G_{z_0}^- = (1 - A)G_{z_0}^+;$$

$$r = 0: v = 0, W_r = 0;$$

$$z = z_M: p = p_0, W_z = -cU/2, G^- = G;$$

$$r = r_N: p = p_0, W_r = -cU/2.$$

As a whole the problem under consideration is rather complicated. As noted above, it describes processes of different scales and is essentially non-linear. Its gas-dynamic, thermophysical and radiative components are closely coupled. A special algorithm, described in details in Ref. [6], has been developed to solve such problems numerically.

3. Analysis of the results

The main attention is given to the peculiarities of the action of a single laser pulses of nanosecond duration at $\lambda = 10.6 \mu\text{m}$ and $\lambda = 1.06 \mu\text{m}$ upon an aluminium target. The spatial-temporal distribution of the laser pulse intensity G is given as the product of two Gaussians (note, that instant $t = 0$ corresponds to the maximum of G):

$$G = G_0 \exp(-r^2/R_\theta^2) \exp(-t^2/\tau^2),$$

$$-\infty < t < \infty,$$

$$G_0 = 10^9 \text{ W/cm}^2, \quad R_\theta = 300 \mu\text{m}, \quad \tau = 30 \text{ ns}.$$

It is assumed that with such a choice of the spatial intensity distribution the main portion of radiation will be absorbed near the beam axis, whereas at the distance $r > R_\theta$ the ambient gas remains cold. Taking this assumption into consideration, the equality $r = R_\theta$ is used in calculations as the minimum size of the initial high temperature plasma region with respect to the variable r . The thickness and temperature of this region for all the computation variants are taken to be constant and equal to $\Delta Z = 50 \mu\text{m}$ and $T = 1 \text{ eV}$, respectively. Note, that chemical composition of the initial high temperature region as well

as of the ambient atmosphere is the same and corresponds to nitrogen. Hence in the present simulation the plasma of ambient gas is considered (without metallic vapour).

The surface absorptivity $A(T_{sr})$ is the important parameter determining the temperature dynamics of the target. As the metal surface is heated (Al target is considered), its absorptivity $A(T_{sr})$ increases by the linear law $A(T_{sr}) = A_0 + B(T_v - T_0)$ from $A_0 = 0.05$ to $A = 0.25$ at $T = T_v$. Here T_{sr} , T_v are the surface temperature and the target evaporation temperature.

The assumed absence of any noticeable evaporation is controlled by comparing the saturated vapour pressure p_s with the plasma pressure at the target surface p_{sr} . In case of intensive surface evaporation (i.e. $p_s \geq p_{sr}$), the consideration of the interaction of the radiation with the surface in the framework of the model Eqs. (1)–(8) becomes incorrect.

3.1. Process evolution

Laser radiation is free to propagate in a cold gas along the z -axis and is absorbed in the high-temperature region near the surface. Laser radiation absorption results in raising the temperature, pressure and ionisation of the medium. Under the pushing-out action of hot layers in the cold gas there appears a shock wave, whose intensity is determined by the state of the high temperature region. As the temperature rises, the role of the intrinsic plasma radiation is also increases. Powerful radiation fluxes determine energy losses in the hottest region of plasma and the heating of the cold gas at rest. Thus the radiation field is superimposed on gas-dynamic processes and changes their evolution. By changing the temperature, density and pressure of the medium, the gas-dynamic motion affects the radiation transfer processes. The strong dependence of the absorption coefficient $\kappa_i = \kappa_i(\rho_i, T, \nu)$ on the density, temperature, frequency of the radiation leads to the variations of the optical density not only under the action of the gas-dynamic flow, but to a greater extent depending on the laser radiation wavelength.

Taking into account that the absorption coefficient of radiation increases as $\sim \lambda^3$, the processes of absorption and plasma development for CO_2 -lasers and Nd:YAG lasers will be essentially different.

3.2. CO_2 -laser action

For CO_2 -laser radiation the high temperature region with $T = 1$ eV, $\Delta Z = 50 \mu\text{m}$ and $\Delta r = R_\theta$ is found to be completely opaque. All the energy of the central part $r \leq R_\theta$ of the laser pulse is released in

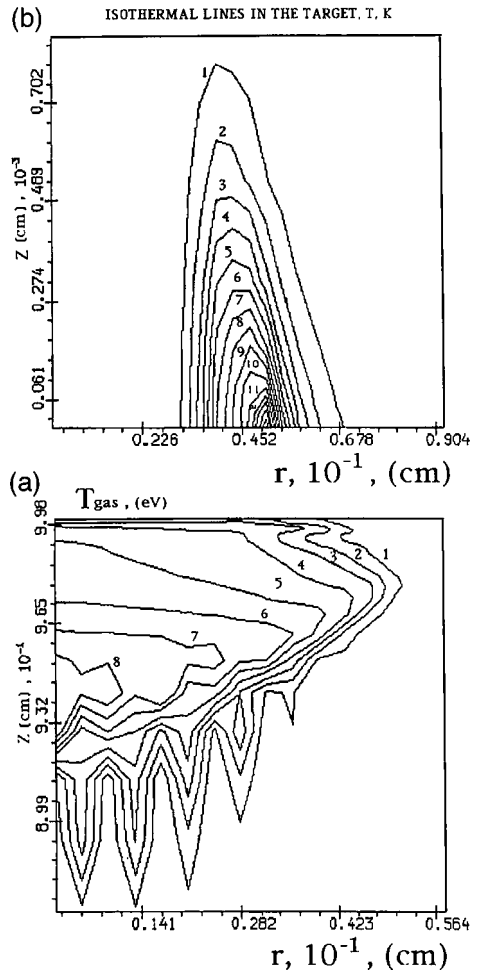


Fig. 1. The temperature fields in the gaseous medium (a) and in the target (b) at the time instant $t = 0$ (i.e. in the middle of the pulse) under the laser action with the $G = 10^9 \text{ W/cm}^2$ at $\lambda = 10.6 \mu\text{m}$ on the initially given plasma layer with $\Delta Z = 50 \mu\text{m}$, $\Delta r = R_\theta = 300 \mu\text{m}$, $T = 1$ eV; ambient atmosphere: nitrogen at normal pressure. (a): curve 1 corresponds to $T_g = 0.53$, 2 – 1.02, 3 – 1.52, 4 – 2.02, 5 – 2.53, 6 – 3.01, 7 – 3.51, 8 – 4.01 eV. (b): curve 1 corresponds to $T = 682$, 2 – 1015, 3 – 1349, 4 – 1682, 5 – 2016, 6 – 2349, 7 – 2683, 8 – 3016, 9 – 3350, 10 – 3683, 11 – 4017, 12 – 4350, 13 – 4684, 14 – 5017, 15 – 5350 K.

the thin plasma layer, heating it rapidly up to 4 eV. The gas becomes completely ionised, the electron concentration being $\sim 2 \times 10^{20} \text{ cm}^{-3}$ exceeds the critical one. The plasma charge composition involves multi-charged ions, $N^{2+} \approx 3 \times 10^{19} \text{ cm}^{-3}$ and, $N^{3+} \approx 2 \times 10^{17} \text{ cm}^{-3}$ being the most typical.

Let us note, that laser radiation is spreading into positive direction of z -axis, and that the target's surface is situated at $z = 1 \text{ cm}$ in Fig. 1a, Fig. 2a,

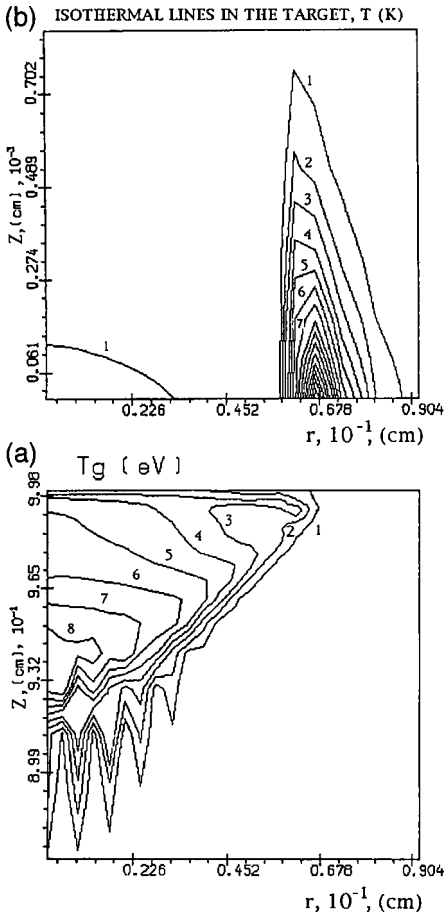


Fig. 2. The temperature fields in the gaseous medium (a) and in the target (b) at the time instant $t=0$ (i.e. in the middle of the pulse) under the laser action with the $G = 10^9 \text{ W/cm}^2$ at $\lambda = 10.6 \mu\text{m}$ on the initially given plasma layer with $\Delta Z = 50 \mu\text{m}$ $\Delta r = 2R_0 = 600 \mu\text{m}$, $T = 1 \text{ eV}$; ambient atmosphere: nitrogen at normal pressure. (a): curve 1 corresponds to $T_g = 0.54$, 2 – 1.06, 3 – 1.57, 4 – 2.09, 5 – 2.6, 6 – 3.11, 7 – 3.63, 8 – 4.14 eV. (b): curve 1 corresponds to $T = 374$, 2 – 399, 3 – 424, 4 – 449, 5 – 474, 6 – 499, 7 – 524, 8 – 549, 9 – 574, 10 – 599, 11 – 624, 12 – 649, 13 – 674, 14 – 700, 15 – 724 K.

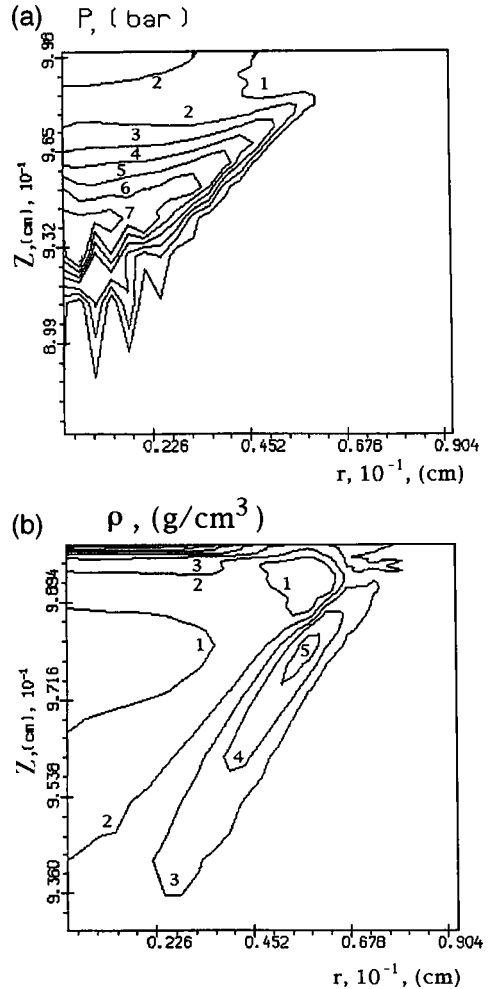


Fig. 3. Distribution of pressure (a) and density (b) in the gas phase for the conditions reported in Fig. 2. (a): curve 1 corresponds to $P = 99$, 2 – 196, 3 – 295, 4 – 435, 5 – 491, 6 – 588, 7 – 686 bar. (b): curve 1 corresponds to $\rho = 6.63 \times 10^{-4}$, 2 – 1.07×10^{-3} , 3 – 1.49×10^{-3} , 4 – 1.9×10^{-3} , 5 – $2.31 \times 10^{-3} \text{ g/cm}^3$.

Fig. 3, Fig. 4a and Fig. 5, and at $z = 0$ in the Figs. 1b, 2b and 4b.

The hottest part of the plasma starts radiating in a wide spectral range just as a black body. The maximum radiation intensity is as great as $2 \times 10^7 \text{ W/cm}^2$ and is in the UV-spectral range $\hbar\nu \in [10-14] \text{ eV}$. Owing to its spectral composition the own plasma radiation exerts a greater influence on the cold gas than the laser radiation does, although the intensity of plasma radiation is lower than that of the laser flux by nearly 2 orders of magnitude. The

UV part of the plasma radiation spectrum is absorbed far before the shock wave.

The jagged form of isothermal and isobar lines (Figs. 2a and 3a) is supposed to be determined by this phenomenon, i.e. intensive radiation transfer from plasma towards ambient cold gas. The mechanism (that may be considered as a typical example of a positive feedback) is the following: the higher is

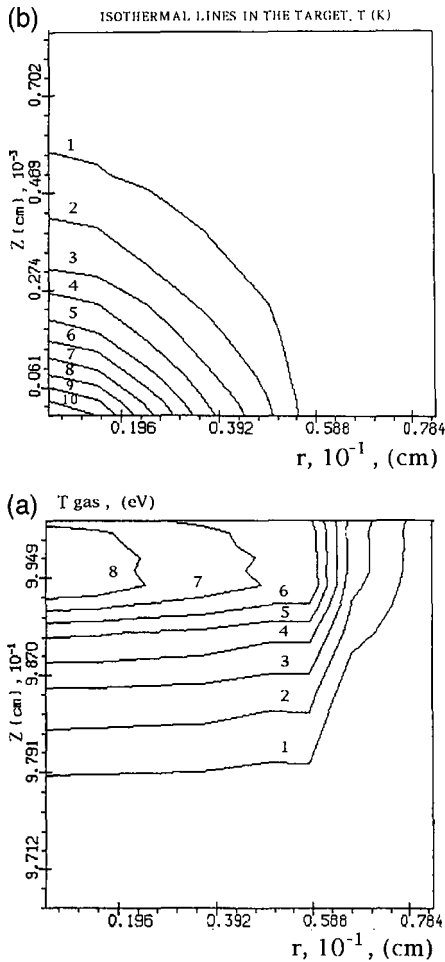


Fig. 4. The temperature fields in the gaseous medium (a) and in the target (b) at the time instant $t = 0$ (i.e. in the middle of the pulse) under the laser action with the $G = 10^9$ W/cm² at $\lambda = 1.06$ μ m on the initially given plasma layer with $\Delta Z = 50$ μ m, $\Delta r = 2R_0 = 600$ μ m, $T = 1$ eV; ambient atmosphere: nitrogen at normal pressure. (a): curve 1 corresponds to $T_g = 0.139$, 2 – 0.248, 3 – 0.358, 4 – 0.467, 5 – 0.576, 6 – 0.685, 7 – 0.794, 8 – 0.903 eV. (b): curve 1 corresponds to $T = 3527$, 2 – 6705, 3 – 9882, 4 – 13060, 5 – 16237, 6 – 19414, 7 – 22592, 8 – 25769, 9 – 28947, 10 – 32124 K.

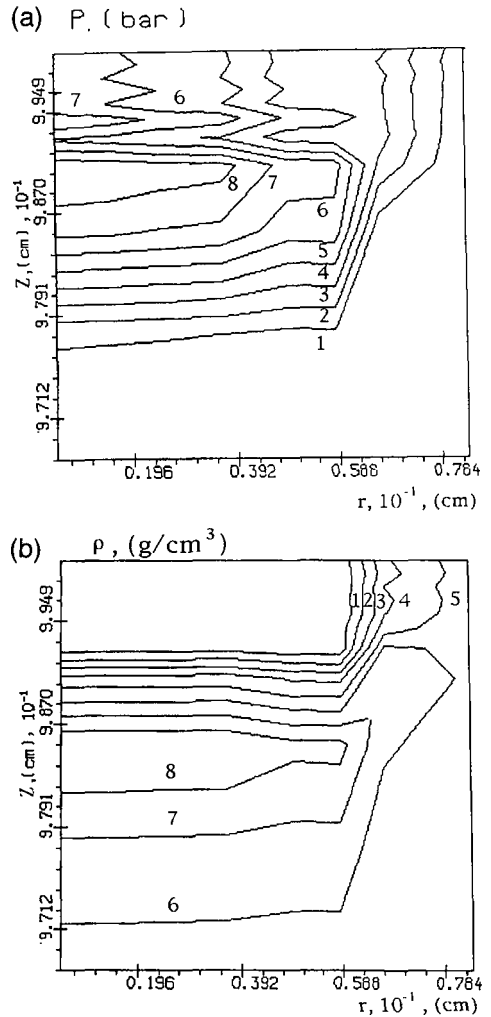


Fig. 5. Distribution of pressure (a) and density (b) in the gas phase for the conditions reported in Fig. 4. (a): curve 1 corresponds to $P = 0.131$, 2 – 0.232, 3 – 0.334, 4 – 0.435, 5 – 0.536, 6 – 0.637, 7 – 0.738, 8 – 0.839 bar. (b): curve 1 corresponds to $\rho = 0.131$, 2 – 0.232, 3 – 0.333, 4 – 0.435, 5 – 0.536, 6 – 0.637, 7 – 0.738, 8 – 0.839 g/cm³.

the temperature of plasma and its absorption coefficient, the larger will be the absorbed laser energy, and therefore the stronger will be the heating of ambient gas (by the plasma self radiation) leading to the increase of its absorptivity. As a result the hot region should quickly spread in the beam direction forming a conical surface (a “spike”, or the “conical rings”). On the other hand, the energy losses from the “spike” in radial direction will increase strongly with its extend. This limits the “spike”

elongation. Hence the formation of the “spike” is determined by the competition of two mechanisms influencing its energy balance: (a) absorption of laser energy in a hot zone, its re-emission resulting in ambient gas heating, and hot zone expansion along the beam; (b) strong energy losses in radial direction from the elongating hot zone. As the confirmation of the above mentioned mechanism two facts could be mentioned: (1) the “spikes” are oriented along the laser beam axis (following the positive feedback between energy absorption, its re-emission and hot zone expansion); (2) the “spikes” exist only at a high enough power density that is required for intensive release of absorbed energy by the plasma: (a) about the middle of laser pulse action when its intensity is the highest (on the contrary, at the beginning and at the end of laser pulse action the isothermal and the isobar lines are smooth); (b) the “spikes” disappear at a certain distance from the beam axis.

Radiation with the intensity $\sim 10^5$ W/cm² is irradiated from the plasma toward the target. Heating the surface by this radiation is found to be insignificant. The surface temperature at $r \approx 0$ does not exceed 500 K.

However, with the chosen size of the initial high temperature region in the gas phase, the target turns out to be unprotected from the action of the periphery part of the laser pulse. The maximum intensity absorbed in the periphery region is as great as 4×10^7 W/cm². As a result the target area marked by the circular band with $320 < r < 570$ μm is heated much more intensively than the central area with $r < R_\phi$, Fig. 1b. In the narrow band $450 < r < 500$ μm the saturated vapour pressure is several times greater than the plasma pressure at the surface, i.e. the condition $p_s \gg p_{sr}$ for the beginning of intensive evaporation is fulfilled.

Giving the radius of high temperature region twice greater, i.e. $\Delta r = 2R_\phi$, also leads to the non-uniform heating of the target, Figs. 2 and 3. The maximum temperature is also attained in the periphery region, but its value does not exceed 800 K. No crucial changes are observed in the behaviour in the gaseous medium, (compare Figs. 1a and 2a).

3.3. Nd:YAG laser action

The action of laser radiation with the wavelength $\lambda = 1.06$ μm under the same initial conditions pro-

duces qualitatively different results. The laser radiation is absorbed only partially in the plasma layer, Figs. 4 and 5. The flux with the intensity $\sim 2 \times 10^8$ W/cm² reaches the surface of the target. The temperature corresponding to the intensive evaporation is quickly attained on the irradiated surface of the target. Note that evaporation and melting are not considered, that is why the temperature field in the target is used mainly to indicate its intensive destruction. In the gaseous medium the maximum temperature is attained at the end of the pulse and is as great as ~ 1.4 eV. Accordingly, the values of radiation fluxes are smaller by two orders of magnitude than for CO₂-laser action. Therefore, in spite of the fact that plasma expansion velocity is 2–3 times lower than for CO₂-laser radiation, in the whole for Nd:YAG laser action the gas-dynamic processes prevail in comparison to the radiative ones.

The developed model works less and less well as the laser wavelength gets shorter since the high temperature layer close to the target surface shields the target surface less and less well from the incident laser radiation. For KrF wavelength for instance the model could be applied probably only for sufficiently higher values of laser power density than the ones considered now. It is possible to expect that the shielding effect of plasma will be reinforced with the intensity of laser radiation.

4. Conclusion

Under the action of a CO₂-laser radiation in the nanosecond range upon Al target the following processes are dominant: (a) strong energy absorption in the thin initial plasma layer which results in its heating up to 4 eV; (b) the plasma charge composition involves multi-charged ions: N^+ , $N^{2+} \approx 3 \times 10^{19}$ cm⁻³ and, $N^{3+} \approx 2 \times 10^{17}$ cm⁻³ being the most typical; (c) the maximum radiation intensity of plasma is as great as 2×10^7 W/cm² and is in the UV-spectral range $\hbar\nu \in [10-14]$ eV; (d) the UV part of the plasma radiation spectrum is absorbed far before the shock wave, forming prominent «tails» of heating; (e) radiation with the intensity $\sim 10^5$ W/cm² is irradiated from the plasma toward the target, which temperature in the centre of action zone does not exceed 500 K; (e) for $\Delta r = R_\phi$: non-uni-

form spatial heating regimes can be realised so that the intensive evaporation proceeds in the region of a narrow circular band beyond the focusing spot radius; (f) for $\Delta r = 2R_0$: maximum surface temperature is also reached in the periphery region, but its values does not exceed 800 K.

Under the same parameters the action of a Nd:YAG laser on the target is mainly destructive: (a) laser radiation is absorbed only partially in the plasma layer, as a result the flux with the intensity $\sim 2 \times 10^8$ W/cm² reaches the surface of the target; (b) the maximum plasma temperature is attained at the end of the pulse and is as great as ~ 1.4 eV; (c) the values of radiation fluxes are smaller by two orders of magnitude and the plasma expansion velocity is 2–3 times lower than for CO₂-laser radiation; (d) the

temperature corresponding to the intensive evaporation is quickly reached on the target's surface.

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