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Optical Properties of Electron Fermi-Gas of Metals at Arbitrary Temperature and Frequency[1]

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Abstract—Optical properties of Fermi-gas of metals are considered at arbitrary temperature ($T \geq \varepsilon_F$). The expressions for temperature and frequency dependences of permittivity are obtained from the solution of the quantum kinetic equation. Frequency and temperature dependences of reflectivity of irradiated surface and volume factor of absorption are determined using Frennel's formulae.

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INTRODUCTION

The energy flow during irradiation of strongly absorbing condensed medium by a concentrated radiation flux (CRF) is partially reflected from the surface and partially absorbed within a thin near-surface layer. The energy absorption has surface or volume nature depending on the irradiation regime (duration of influence, radiation wave length) and optical and thermophysical properties of the processed materials. The deposited energy is then expended for heating, melting and evaporation of the target.

Systematic investigation of all phenomena developing in the CRF affecting zone is necessary for formulation of the basic requirements to CRF sources and to determine the optimum regimes of influence. For example, laser treatment of opaque materials requires such optical characteristics as surface reflectivity R[%] or absorptance $A = 1 - R$ and also volume absorptance α [cm^{-1}]. For a wide class of materials including metals, there is an extensive information on frequency dependence of optical characteristics [1–3] measured at a fixed (usually room) temperature. Temperature dependences for the majority of materials even in the low temperature region are determined insufficiently precisely. So the temperature dependence of absorptance $A(T)$ is known to be linear for the majority of metals at temperatures below melting temperature, $A(T) = a + bT$, where a, b are certain coefficients. The reflectivity and absorptance are usually assumed to be temperature independent in the region close to or exceeding the melting point, so its average or average-integral values within the considered temperature range are used in estimations [4].

Such approach is not much suitable for mathematical modeling of treatment of metals by ultrashort and high-power laser pulses when the energy of radiation is transferred directly to electrons and strongly non-equilibrium region with hot electrons and cold lattice is formed in solid. In addition, short period of time ($t \sim 10^{-12}$–10^{-15}) s is required for heating of the electronic subsystem to temperatures T_e comparable to or exceeding the Fermi energy E_F. Electrons have Fermi distribution with temperature in the case of $T_e < E_F$. Degeneration is removed at $T_e \cong E_F$ and electrons obtain Maxwell distribution for $T_e > E_F$. Transition through $T_e \sim E_F$ temperature is connected with change of electron-electron and electron–phonon interactions mechanisms that lead to qualitative changes in optical and thermo-physical characteristics of solid [5].

Here, we present an attempt of calculation of the temperature and frequency dependences of optical characteristics of metals in a wide range of frequencies ($\hbar\omega$ = 0.1–10.0 eV) and temperatures (T_e = 0.024–50 eV). To achieve this, we use the longitudinal permittivity $\varepsilon^l = \varepsilon^l(\omega, T)$ obtained from the solution of quantum kinetic equation.

1. THE THEORETICAL ANALYSIS

1.1. Reflectivity and Volume Absorptance

It is possible to express all linear (macroscopic) optical characteristics of plasma including real coefficients of absorptance α, of reflection R and complex refraction index $N = n + i\kappa$ in terms of its permittivity [6]. By definition, the complex refraction index N is equal to:

$$N = n + i\kappa = \sqrt{\varepsilon^l}, \quad (1)$$

where n and κ are the optical constants representing the real and imaginary parts of refraction index, ε^l is the longitudinal permittivity. Since permittivity is also a

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complex quantity $\varepsilon^l = \varepsilon_1^l + i\varepsilon_2^l$ one can equate their real and imaginary parts and obtain the following system of equations:

$$\varepsilon_1^l = n^2 - \kappa^2, \quad \varepsilon_2^l = 2n\kappa. \tag{2}$$

The solution of the system (2) gives the expressions for n and κ:

$$n = 2^{-1/2}\{\varepsilon_1^l + [(\varepsilon_1^l)^2 + (\varepsilon_2^l)^2]^{1/2}\}^{1/2}, \tag{3}$$

$$\kappa = 2^{-1/2}\{-\varepsilon_1^l + [(\varepsilon_1^l)^2 + (\varepsilon_2^l)^2]^{1/2}\}^{1/2}. \tag{4}$$

The reflectivity R and absorptance A of irradiated surface of infinitely thick plasma layer at normal falling are expressed according to classical Fresnel formula [7] as

$$R = \frac{(n-1)^2 + \kappa^2}{(n+1)^2 + \kappa^2}, \quad A = 1 - R = \frac{4n}{(n+1) + \kappa^2}. \tag{5}$$

The volume absorptance coefficient α is thus equal to [6]

$$\alpha = \frac{2\kappa\omega}{c} = \frac{4\pi\kappa}{\lambda}, \tag{6}$$

where ω is the frequency of irradiation, λ is the wave length, c is the velocity of light.

The values of R, A, and α for low temperatures are usually determined from experimental data and then are used to obtain optical constants n and κ [6–8].

Generally speaking, the values of n, κ, ε^l for electronic plasma of metals are functions of radiation frequency ω and temperature T:

$$n = n(\omega, T), \quad \kappa = \kappa(\omega, T), \quad \varepsilon^l = \varepsilon^l(\omega, T)$$

Thus, all macroscopic optical properties of metals and their frequency and temperature dependences can be expressed solely in terms of longitudinal permittivity $\varepsilon^l = \varepsilon^l(\omega, T)$.

1.2. Permittivity of Degenerated Electronic Plasma of Metal $\varepsilon(\omega, \mathbf{k})$

It is known from the theory of electromagnetic field that the permittivity of free electron gas $\varepsilon(\omega, \mathbf{k})$ depends on frequency ω (so-called time or frequency dispersion) and wave vector $\mathbf{k}$ (the spatial dispersion). In the presence of space dispersion, i.e., depending on the vector $\mathbf{k}$, the permittivity is a tensor value $\varepsilon_{ij}(\omega, \mathbf{k})$ even in isotropic medium. The tensor $\varepsilon_{ij}(\omega, \mathbf{k})$ is characterized by two scalar functions—ε^l and ε^t, so-called longitudinal and transversal permittivity accordingly which depend on independent variables—frequency ω and wave vector modulus k, $\varepsilon^l = \varepsilon^l(\omega, k)$, $\varepsilon^t = \varepsilon^t(\omega, k)$. Preferential direction disappears at $\mathbf{k} \longrightarrow 0$ and the tensor $\varepsilon_{ij}(\omega, \mathbf{k})$ is reduced to $\varepsilon(\omega)\delta_{ij}$, where $\varepsilon(\omega)$ is the usual scalar permittivity which takes into account only frequency dispersion. The limiting values of functions ε^l and ε^{tr} also become equal to $\varepsilon^l = \varepsilon^l(\omega, 0) = \varepsilon^t = \varepsilon^t(\omega, 0) = \varepsilon(\omega)$.

Generally speaking, the tensor $\varepsilon_{ij}(\omega, \mathbf{k})$ is a complex function of real variables ω and $\mathbf{k}$. Scalar functions ε^l and ε^t are also complex functions of frequency ω and wave vector modulus k.

$$\begin{aligned} \varepsilon^l(\omega, k) &= \varepsilon_1^l(\omega, k) + i\varepsilon_2^l(\omega, k), \\ \varepsilon^t(\omega, k) &= \varepsilon_1^t(\omega, k) + i\varepsilon_2^t(\omega, k). \end{aligned} \tag{7}$$

The presence of time and spatial dispersion in the longitudinal permittivity $\varepsilon^l(\omega, k)$ allows determining its frequency and temperature dependences and corresponding dependences of refractivity and absorptance.

To determine the longitudinal permittivity in arbitrary temperature range, it was supposed that the transition from strong degeneration $\xi = T_e/E_F \ll 1$ to Boltzmann's case $\xi = T_e/E_F \gg 1$ occurs smoothly. The temperature dependence of the chemical potential [6] was used for smoother transition

$$\begin{aligned} \mu(T_e) &\cong E_F\left(1 - \frac{\pi^2}{3!}\xi^2 + \frac{\pi^4}{3!4!}\xi^4\right) = E_F D(\xi), \\ D(\xi) &= \left(1 - \frac{\pi^2}{3!}\xi^2 + \frac{\pi^4}{3!4!}\xi^4\right). \end{aligned} \tag{8}$$

At low temperatures when $\mu(T_e)$ is close to E_F (exact equality $\mu(T_e) = E_F$ is reached at absolute zero), collective electrons in metal obey the Fermi–Dirac statistics $f(E) = (\exp(x - \eta) + 1)^{-1}$, where $x = E/T_e$, $\eta = \mu(T_e)/T_e$, E is the energy of electron. The distribution function $f(E)$ is practically close to Boltzmann's one $f(E) = \exp(\eta - x)$ for high values of energy E when $x - \eta \gg 1$.

The overall longitudinal permittivity is suggested to be defined in the form of the sum of two components providing smooth transition from degenerated electron gas to Maxwell's plasma

$$\begin{aligned} \varepsilon^l = \varepsilon^l(\omega, T) &= D(\xi)\varepsilon_F^l(\omega, T) \\ &+ (1 - D(\xi))\varepsilon_M^l(\omega, T). \end{aligned} \tag{9}$$

1.3. Quantum Kinetic Equation

The calculation of permittivity $\varepsilon(\omega, \mathbf{k})$ in the general case of arbitrary values of vector $\mathbf{k}$ (with essential effect of spatial dispersion) requires application of quantum kinetic equation [9] which for collisionless plasma has the form:

$$\frac{\partial(\delta f)}{\partial t} + \mathbf{v}\frac{\partial(\delta f)}{\partial \mathbf{r}} = e\mathbf{E}\frac{\partial(\delta f_0)}{\partial \mathbf{p}}, \tag{10}$$

where $f(\mathbf{p}) = f_0 + \delta f(p)$ is the electron distribution function in the momentum space, f_0 is the stationary isotropic and space-homogeneous distribution function unperturbed by the field, δf is the variation of the distribution function under the influence of the field.

The longitudinal part of permittivity $\varepsilon^l(\omega, k)$ for the collisionless plasma is determined from the solution of the quantum kinetic equation (10) and has the form [9]

$$\varepsilon^l(\omega, k) = 1 - \frac{4\pi e^2}{k^2}\int_{-\infty}^{\infty} \mathbf{k}\frac{\partial f(\mathbf{p})}{\partial \mathbf{p}}\frac{d^3p}{\mathbf{kv} - \omega - i0}. \quad (11)$$

The derived expression has a critical point (pole) in the bottom complex half plane. The singularity $\frac{1}{\mathbf{kv} - \omega}$ is usually considered [9, 10] as $\frac{1}{\mathbf{kv} - (\omega + i0)}$, i.e., the value ω is represented in the form of $\omega + i0$ with infinitesimal positive imaginary part. After a change of the integration variable the integral (11) is written in the form

$$\int_{-\infty}^{\infty} \frac{f(z)}{z - i\delta}dz, \quad \delta > 0.$$

The path of integration in the complex plane runs under the point $z = i\delta$ that at $\delta \longrightarrow 0$ is equivalent to integration along the real axis with inclusion of the pole $z = 0$ over an infinitesimal semicircle. Contribution to integral from this contour is defined by the half residue of integrand

$$\int_{-\infty}^{\infty} \frac{f(z)}{z - i\delta}dz = \int_{-\infty}^{\infty} \frac{f(z)}{z}dz + i\pi f(0). \quad (12)$$

The integral in the right part of equation (12) is the limiting value of an integral of Cauchy type.

The quantum analogue of the classical kinetic equation [10] is used for description of electromagnetic properties of the quantum plasma to which electronic plasma of metals refers to. The equation for the quantum distribution function $f(\mathbf{p})$ dependent on the kinematic momentum $\mathbf{p}$, for a small deviation from the equilibrium homogeneous condition $f(\mathbf{p}, \mathbf{r}, t) = f_0(\mathbf{p}) + \delta f(\mathbf{p}, \mathbf{r}, t)$, [10] has form:

$$\frac{\partial(\delta f)}{\partial t} + \mathbf{v}\frac{\partial(\delta f)}{\partial \mathbf{r}} + e\mathbf{E}\frac{\partial(\delta f_0)}{\partial \mathbf{p}} = \frac{e}{(2\pi)^3}\frac{i}{\hbar}\int e^{i\tau(p' - p)}$$
$$\times f_0(\mathbf{p}')\left\{\left[\frac{\partial\varphi}{\partial\mathbf{r}}\hbar\boldsymbol{\tau} - \varphi\left(\mathbf{r} + \frac{\hbar\boldsymbol{\tau}}{2}\right) + \varphi\left(\mathbf{r} - \frac{\hbar\boldsymbol{\tau}}{2}\right)\right]\right. \quad (13)$$
$$\left. - \frac{\mathbf{v}'}{c}\left[\left(\hbar\boldsymbol{\tau}\frac{\partial\mathbf{A}}{\partial\mathbf{r}}\right) - \mathbf{A}\left(\mathbf{r} + \frac{\hbar\boldsymbol{\tau}}{2}\right) + \mathbf{A}\left(\mathbf{r} - \frac{\hbar\boldsymbol{\tau}}{2}\right)\right]\right\}d\boldsymbol{\tau}d\mathbf{p}',$$

where f_0 is the stationary isotropic and space-homogeneous momentum distribution function of electrons unperturbed by the field, δf is its variation under the influence of the field, $\mathbf{E}$, φ, $\mathbf{A}$ are electric field strength vector, scalar and vector potentials, $\boldsymbol{\tau} = \mathbf{p}^{-1}$.

The equation (13) changes into the kinetic equation (10) at the classical limit $\hbar \longrightarrow 0$. The function $f(\mathbf{p})$ for degenerated electron gas takes the form of Fermi distribution:

$$f_{\mathrm{F}}(\mathbf{p}) = \frac{2n(\mathbf{p})}{(2\pi\hbar)^3},$$

where $\frac{2d^3p}{(2\pi\hbar)^3}$ is the number of conditions within the momentum space element d^3p with two values of spin projection, $n(\mathbf{p})$ is the number of filled quantum electron states with specified values of momentum and spin projections. $n(\mathbf{p}) = 1$ in the case of complete degeneration ($T = 0$) and distribution function takes the form:

$$f_F(\mathbf{p}) = \begin{cases} \frac{2}{(2\pi\hbar)^3}, & p < p_F = (3\pi^2)^{1/3}\hbar N_e^{1/3} \\ 0, & p > p_F. \end{cases} \quad (14)$$

The expression for longitudinal permittivity $\varepsilon^l(\omega, \mathbf{k})$ of completely degenerate electron gas with distribution function (14) is obtained in [11] from the solution of the quantum kinetic equation (13):

$$\varepsilon^l(\omega, \mathbf{k}) = 1 - \frac{4\pi e^2}{\hbar k^2}$$
$$\times \int \frac{\left[n\left(\mathbf{p} + \frac{\hbar\mathbf{k}}{2}\right) - n\left(\mathbf{p} - \frac{\hbar\mathbf{k}}{2}\right)\right]}{\mathbf{kv} - (\omega + i0)}\frac{2d^3p}{(2\pi\hbar)^3}. \quad (15)$$

A more general expression for permittivity of degenerate plasma is obtained at $T \neq 0$. Elementary but rather tedious integration of equation (15) leads to the following result [12]:

$$\varepsilon^l(\omega, k) = 1 - \frac{4\pi m e^2 N_e}{\hbar k^3 p_{Te} F_{1/2}(\xi)}$$
$$\times \int_{-\infty}^{\infty} \ln\left[\frac{1 + \exp\left(\eta(\xi) - \left(\frac{p}{p_{Te}} - \frac{\hbar k}{2p_{Te}}\right)^2\right)}{1 + \exp\left(\eta(\xi) - \left(\frac{p}{p_{Te}} + \frac{\hbar k}{2p_{Te}}\right)^2\right)}\right] \quad (16)$$
$$\times \frac{dp}{kp/m - (\omega + i0)},$$

where ϑ_{Te}, p_{Te} are the average values of thermal velocity and electron momentum, k is the average value of the modulus of the wave vector $\mathbf{k}$.

The integral in the equation (16) is a limiting value of integral of Cauchy type due to the presence of

$\frac{1}{\mathbf{kv} - (\omega + i0)}$ singularity. The integration path runs in the plane of the complex variable $(\omega + i0)$ along the real axis with inclusion of the point $p = m\omega/k$.

1.4. Imaginary Part of the Longitudinal Permittivity $\varepsilon_2^l(\omega, \mathbf{k})$

The imaginary part $\varepsilon_2^l(\omega, \mathbf{k})$ of the longitudinal permittivity $\varepsilon^l(\omega, \mathbf{k})$ is defined in (16) by a half residue in the point $p = m\omega/k$. We separate it using the expression (12) and obtain:

$$\varepsilon_2^l(\omega, k) = \frac{4\pi^2 e^2 m N_e}{\hbar k^3 p_T F_{1/2}(\xi)} \times \ln\left\{\frac{1 + \exp\left(\eta(\xi) - \left(\frac{p}{p_{Te}} - \frac{\hbar k}{2p_{Te}}\right)^2\right)}{1 + \exp\left(\eta(\xi) - \left(\frac{p}{p_{Te}} + \frac{\hbar k}{2p_{Te}}\right)^2\right)}\right\}. \quad (17)$$

The imaginary part of permittivity $\varepsilon_{2,F}^l(\omega, k)$ in the case of degenerate electron Fermi gas with account of expressions $p = m\omega/k$, $k_F = p_F/\hbar$, $p_F = m v_F = (2m\varepsilon_F)^{1/2}$, $v_F = (2\varepsilon_F/m)^{1/2}$ is written as:

$$\varepsilon_{2,F}^l(\omega, k) = 3\pi\frac{T_e(\hbar\omega_{Le})^2}{(\hbar\omega)^3} \times \ln\left\{\frac{1 + \exp\left(\eta(\xi) - \left(\frac{\omega}{k_F v_{Te}} - \frac{\hbar k_F}{2m v_{Te}}\right)^2\right)}{1 + \exp\left(\eta(\xi) - \left(\frac{\omega}{k_F v_{Te}} + \frac{\hbar k_F}{2m v_{Te}}\right)^2\right)}\right\}, \quad (18)$$

where $\omega_{Le} = \frac{v_{Te}}{d_e} = \left(\frac{4\pi e^2 N_e}{m}\right)^{1/2}$, $d_e = \left(\frac{T_e}{4\pi e^2 N_e}\right)^{1/2}$ is the plasma or Langmuir frequency and Debye radius for electrons accordingly, $F_{k+1/2} = \int_0^\infty \frac{x^{k+1/2}}{\exp(x-\eta)+1}dx$ is the Fermi's integral. The values of $F_{1/2}$ and $\eta(\xi)$ are approximated by expressions [5]:

$$F_{1/2} \cong \frac{2}{3}\xi^{-3/2}, \quad \eta(\xi) = \xi^{-1} + \ln\frac{4/3\pi^{1/2}}{\xi^{3/2} + 4/3\pi^{1/2}}.$$

The imaginary part of permittivity for Maxwell plasma with account of $k_{Te} = p_{Te}/\hbar = \hbar^{-1}(2mT_e)^{1/2}$, $p_{Te} = m v_{Te} = (2mT_e)^{1/2}$, $\vartheta_{Te} = (2T_e/m)^{1/2}$ can be written as:

$$\varepsilon_{2,M}^l(\omega, k) = 2\pi\frac{T_e(\hbar\omega_{Le})^2}{(\hbar\omega)^3} \times \ln\left\{\frac{1 + \exp\left(\eta(\xi) - \left(\frac{\omega}{k_{Te} v_{Te}} - \frac{\hbar k_{Te}}{2m v_{Te}}\right)^2\right)}{1 + \exp\left(\eta(\xi) - \left(\frac{\omega}{k_{Te} v_{Te}} + \frac{\hbar k_{Te}}{2m v_{Te}}\right)^2\right)}\right\}. \quad (19)$$

The expressions (18)–(19) in two limiting cases of low $T_e \longrightarrow 0$ and high $T_e \gg \varepsilon_F$ change into two well-known relationships. The expression (18) changes at $T_e \longrightarrow 0$ into the formula obtained in [13] for imaginary part of permittivity $\varepsilon_2^l(\omega, k)$ of degenerate electron Fermi-gas:

$$\varepsilon_{2,F}^l(\omega, k) = \frac{3\pi}{2}\frac{\omega_{Le}^2\omega}{(k v_F)^3} \quad \text{at} \quad |\omega| < k\vartheta_F.$$

In other limiting case of sufficient for degeneration removal high temperatures $\xi \gg 1$ and $h \longrightarrow 0$, relationship (19) matches a known expression $\varepsilon_{2,M}^l(\omega, k)$ for classical electron plasma [9]:

$$\varepsilon_2^l(\omega, k) = \left(\frac{\pi}{2}\right)^{1/2}\frac{\omega\omega_{Le}^3}{(k v_{Te})^3}\exp\left(-\frac{1}{2}\left(\frac{\omega}{k v_{Te}}\right)^2\right).$$

Thus, the imaginary part of permittivity of degenerated electron gas $\varepsilon_{2,F}^l(\omega, k)$ and Maxwell plasma $\varepsilon_{2,M}^l(\omega, k)$ can be written in the form of frequency and temperature dependences

$$\varepsilon_{2,F}^l(\omega, T_e) = C(\hbar\omega)T_e\frac{(\hbar\omega_{Le})^2}{(\hbar\omega)^3} \times \ln\left\{\frac{1 + \exp\left(\eta(\xi) - \frac{E_F}{4T_e}\left(\frac{\hbar\omega}{E_F} - 1\right)^2\right)}{1 + \exp\left(\eta(\xi) - \frac{E_F}{4T_e}\left(\frac{\hbar\omega}{E_F} + 1\right)^2\right)}\right\},$$

$$\varepsilon_{2,M}^l(\omega, T_e) = C(\hbar\omega)T_e\frac{(\hbar\omega_{Le})^2}{(\hbar\omega)^3} \times \ln\left\{\frac{1 + \exp\left(\eta(\xi) - \frac{1}{4}\left(\frac{\hbar\omega}{T_e} - 1\right)^2\right)}{1 + \exp\left(\eta(\xi) - \frac{1}{4}\left(\frac{\hbar\omega}{T_e} + 1\right)^2\right)}\right\}.$$

The expression for the total imaginary part of permittivity ε_2^l, according to (9) is presented in the form explicitly dependent on the radiation frequency to and electron temperature T_e:

$$\varepsilon_2^l(\omega, T_e) = D(\xi)\varepsilon_{2,F}^l(\omega, T_e) + (1-D(\xi))\varepsilon_{2,M}^l(\omega, T_e) = \left(C\frac{T_e(\hbar\omega_{Le})^2}{(\hbar\omega)^3}\right) \times \left\{ D(\xi)\ln\left[\frac{1+\exp\left(\eta(\xi)-\frac{1}{4\xi}\left(\frac{\hbar\omega}{E_F}-1\right)^2\right)}{1+\exp\left(\eta(\xi)-\frac{1}{4\xi}\left(\frac{\hbar\omega}{E_F}+1\right)^2\right)}\right] + (1-D(\xi))\ln\left[\frac{1+\exp\left(\eta(\xi)-\frac{1}{4}\left(\frac{\hbar\omega}{T_e}-1\right)^2\right)}{1+\exp\left(\eta(\xi)-\frac{1}{4}\left(\frac{\hbar\omega}{T_e}+1\right)^2\right)}\right]\right\}, \quad (20)$$

where the value $D(\xi) = \left(1-\frac{\pi^2}{3!}\xi^2+\frac{\pi^4}{3!4!}\xi^4\right)$, $C = C(\hbar\omega)$ weakly varies with frequency, $C = 1–2\pi$ within the frequency range $\hbar\omega = 0.1–100$ eV.

1.5. The Real Part of Longitudinal Permittivity $\varepsilon_1^l(\omega, \mathbf{k})$

Analytical determination of real part of permittivity $\varepsilon_1^l = \varepsilon_1^l(\omega, \mathbf{k})$ by means of the equation (16) is possible only for two limiting cases [12]: for high $\omega/kv_{Te} \gg 1$ and low $\omega/kv_{Te} \ll 1$ frequencies.

1.5.1. High-frequency approximation $\varepsilon_1^{l,h}(\omega, k)$. At high frequencies $\omega/kv_{Te} \gg 1$ the integrand (16) can be expanded in a Taylor series witch integration gives the approximation for the real part of permittivity $\varepsilon_1^{l,h}(\omega, k)$:

$$\varepsilon_1^{l,h}(\omega, k) = 1-\frac{4\pi e^2 N_e}{kp_{Te}}\frac{1}{\omega F_{1/2}} \times \sum_{j=0}^{\infty}\left[\left(\frac{kv_{Te}}{\omega}\right)^{2j+1}F_{(2j+1)/2}\right]. \quad (21)$$

We preserve the first three terms of expansion in (21) and obtain:

$$\varepsilon_1^{l,h}(\omega, T_e) = 1-\frac{\omega_{Le}^2}{kv_{Te}}\frac{1}{\omega}\left(\left(\frac{kv_{Te}}{\omega}\right) + \left(\frac{kv_{Te}}{\omega}\right)^3\frac{F_{3/2}}{F_{1/2}} + \left(\frac{kv_{Te}}{\omega}\right)^3\frac{F_{5/2}}{F_{1/2}} + \ldots\right).$$

For Fermi component

$$\varepsilon_{1,F}^{l,h}(\omega, T_e) = 1-\left(\frac{\omega_{Le}}{\omega}\right)^2\left(1+\left(\frac{k_F v_{Te}}{\omega}\right)^2\frac{F_{3/2}}{F_{1/2}} + \left(\frac{k_F v_{Te}}{\omega}\right)^4\frac{F_{5/2}}{F_{1/2}} + \ldots\right).$$

Taking into account values of Fermi integrals $F_{k+1/2}$, and average electron energy $\langle E\rangle = T_e\frac{F_{3/2}}{F_{1/2}}$ approximation for $\varepsilon_{1,F}^{l,h}$ is written as

$$\varepsilon_{1,F}^{l,h} \approx 1-\left(\frac{\hbar\omega_{Le}}{\hbar\omega}\right)^2\left[1+\frac{4E_F\langle E\rangle}{(\hbar\omega)^2}+\frac{(4E_F)^2\langle E\rangle^2}{(\hbar\omega)^4}\right]. \quad (22)$$

For Maxwell plasma $\xi > 1$

$$\varepsilon_{1,M}^{l,h}(\omega, T_e) = 1-\left(\frac{\hbar\omega_{Le}}{\hbar\omega}\right)^2\left(1+\left(\frac{k_{T_e}v_{Te}}{\omega}\right)^2+\left(\frac{k_{T_e}v_{Te}}{\omega}\right)^4\right) = 1-\left(\frac{\hbar\omega_{Le}}{\hbar\omega}\right)^2\left(1+\left(\frac{2T_e^2}{\hbar\omega}\right)^2+\left(\frac{2T_e^2}{\hbar\omega}\right)^4\right). \quad (23)$$

1.5.2. Low-frequency approximation $\varepsilon_1^{l,s}$. A change of variables $y = x + \omega/kv_T$ is preformed in integral (16) for low frequencies $\omega/kv_T \ll 1$. After expansion in series and integration within x we obtain:

$$\varepsilon_1^{l,s} = 1-\frac{4\pi e^2 N_e}{k^2 p_{Te}^2}\frac{m}{F_{1/2}} \times \left(F_{-1/2}+\sum_{j=0}^{\infty}(-1)^j\frac{2^{2j+1}j!}{(2j+1)!}\frac{2+j}{1+j}\left(\frac{\omega}{kv_T}\right)^{2(j+1)} \times \int_0^{\infty}\frac{f^{(j)}(x)}{\sqrt{x}}dx\right), \quad (24)$$

where $f^{(j)}(x)$ is the j-th derivative of Fermi distribution function. Preserving only first two components $j = 0, 1$

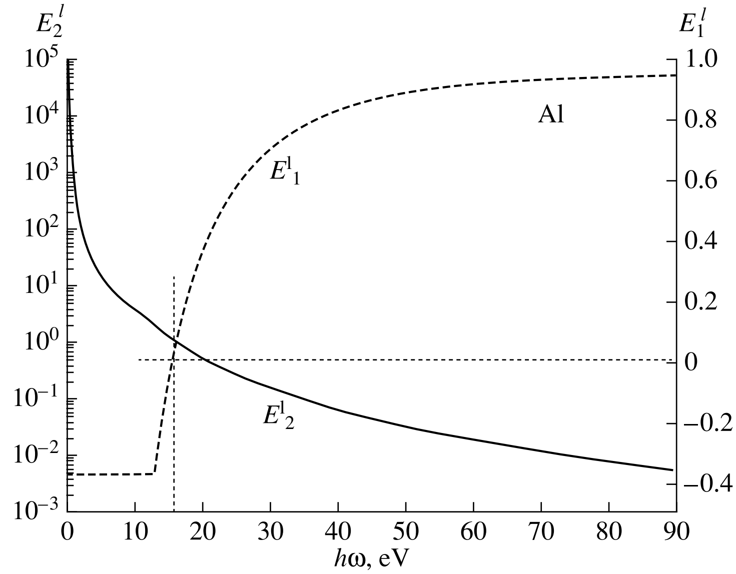

Fig. 1. Frequency dependence real $E_1^l(\omega)$ and imaginary part $E_2^l(\omega)$ of longitudinal inductivity at temperature $T = 290$ K.

in series (24) we write:

$$\varepsilon_1^{l,s}(\omega, k) = 1 - \frac{4\pi e^2 N_e}{k^2 p_{T_e}^2}\frac{m}{F_{1/2}}$$

$$\times\left(F_{-1/2} + 4\left(\frac{\omega}{kv_T}\right)^2\int_0^\infty\frac{f^{(0)}(x)}{\sqrt{x}}dx - 2\left(\frac{\omega}{kv_T}\right)^4\int_0^\infty\frac{f^{(1)}(x)}{\sqrt{x}}dx\right).$$

At $\xi < 1$ for Fermi distribution we obtain:

$$\varepsilon_{1,F}^{l,s}(\omega, k) \approx 1 - \frac{4\pi e^2 N_e}{k_F^2 p_{T_e}^2}\frac{F_{-1/2}}{F_{1/2}}$$

$$\times\left[1 + 4\left(\frac{\omega}{k_F v_T}\right)^2 + 2\left(\frac{\omega}{k_F v_T}\right)^4\right].$$

Considering that $\int_0^\infty\frac{f^{(0)}(x)}{\sqrt{x}}dx = \int_0^\infty\frac{x^{-1/2}dx}{\exp(x-\eta)+1} =$ $F_{-1/2}$ and $\frac{F_{-1/2}}{F_{1/2}} = \frac{2\xi}{(\xi^2 + (2/3)^2)^{1/2}}$, the low-frequency approximation for Fermi components take the form:

$$\varepsilon_{1,F}^{l,s}(\omega, k) \approx 1 - \frac{(\hbar\omega_{Le})^2}{E_F^2}\frac{1}{(\xi^2 + (4/9))^{1/2}}$$
$$\times\left(1 + \frac{(\hbar\omega)^2}{E_F T_e} + \frac{1}{8}\frac{(\hbar\omega)^4}{(E_F T_e)^2}\right). \quad (25)$$

Corresponding value for Maxwell component $\xi > 1$ is written as:

$$\varepsilon_{1,M}^{l,s}(\omega, k) \approx 1 - \frac{1}{2}\left(\frac{\hbar\omega_{Le}}{T_e}\right)^2$$
$$\times\left[1 + \left(\frac{\hbar\omega}{T_e}\right)^2 + \frac{1}{8}\left(\frac{\hbar\omega}{T_e}\right)^4\right]. \quad (26)$$

Taking into account the expressions (22), (25), the real part of permittivity of degenerated electron gas $\varepsilon_{1,F}^l$, we can be presented (25) in the form of lacing of high-frequency $\varepsilon_{1,F}^{l,h}(\omega, T_e)$ and low-frequency $\varepsilon_{1,F}^{l,s}(\omega, T_e)$ approximations. The high-frequency $\varepsilon_{1,F}^{l,h}(\omega, T_e)$ approximation satisfies the condition $\omega/kv_{Te} \gg 1$ and is used in low-temperature region. Low-frequency approximation $\varepsilon_{1,F}^{l,s}(\omega, T_e)$ satisfies the condition $\omega/kv_{Te} \ll 1$ and is applied to high temperatures. The lacing is carried out in the point of intersection of the curves $\varepsilon_{1,F}^{l,h}(\omega, T_e)$ and $\varepsilon_{1,F}^{l,s}(\omega, T_e)$ where the permittivity transits from $\varepsilon_{1,F}^{l,h}(\omega, T_e)$ curve to $\varepsilon_{1,F}^{l,s}(\omega, T_e)$ curve.

The same method of lacing of high-frequency $\varepsilon_{1,M}^{l,h}(\omega, T_e)$ and low-frequency $\varepsilon_{1,M}^{l,s}(\omega, T_e)$ approximations is used to determine the real part of permittivity for Maxwell plasma $\varepsilon_{1,M}^l(\omega, T_e)$. Using the obtained expressions for $\varepsilon_{1,F}^l$ and $\varepsilon_{1,M}^l(\omega, T_e)$ real part of permittivity $\varepsilon_1^{l,h}(\omega, k)$ is finally presented as:

$$\varepsilon_1^l(\omega, T) = D(\xi)\varepsilon_{1,F}^l(\omega, T)$$
$$+ (1 - D(\xi))\varepsilon_{1,M}^l(\omega, T). \quad (27)$$

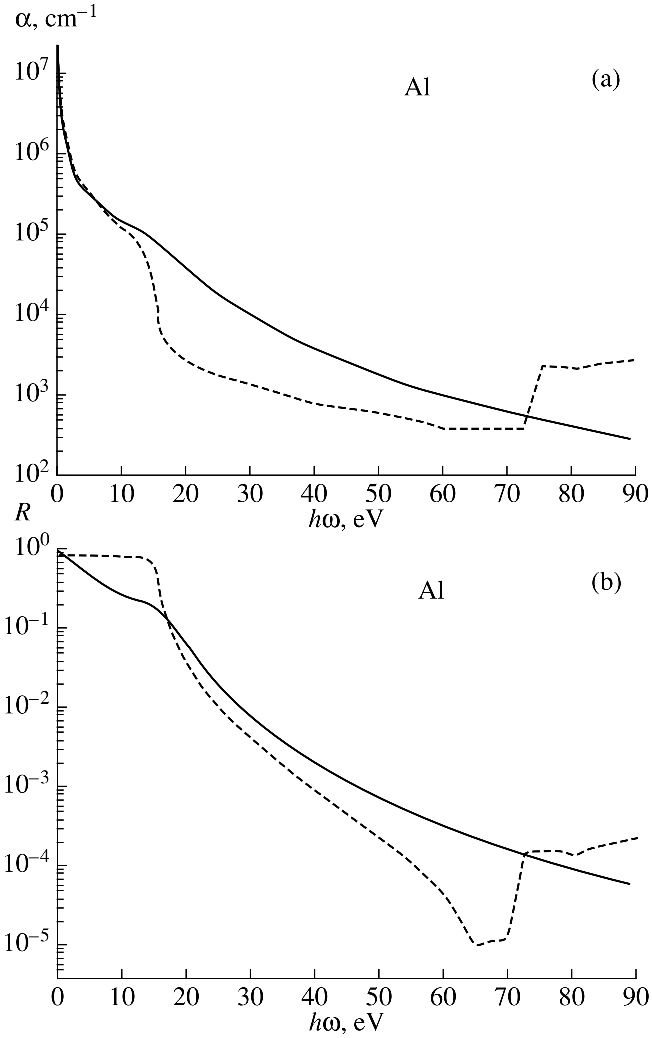

Fig. 2. Frequency dependence of (a) volume absorptance $\alpha(\omega)$, (b) surface reflectivity $R(\omega)$ at temperature $T = 290$ K. Solid lines—calculated curves, dashed lines—reference data.

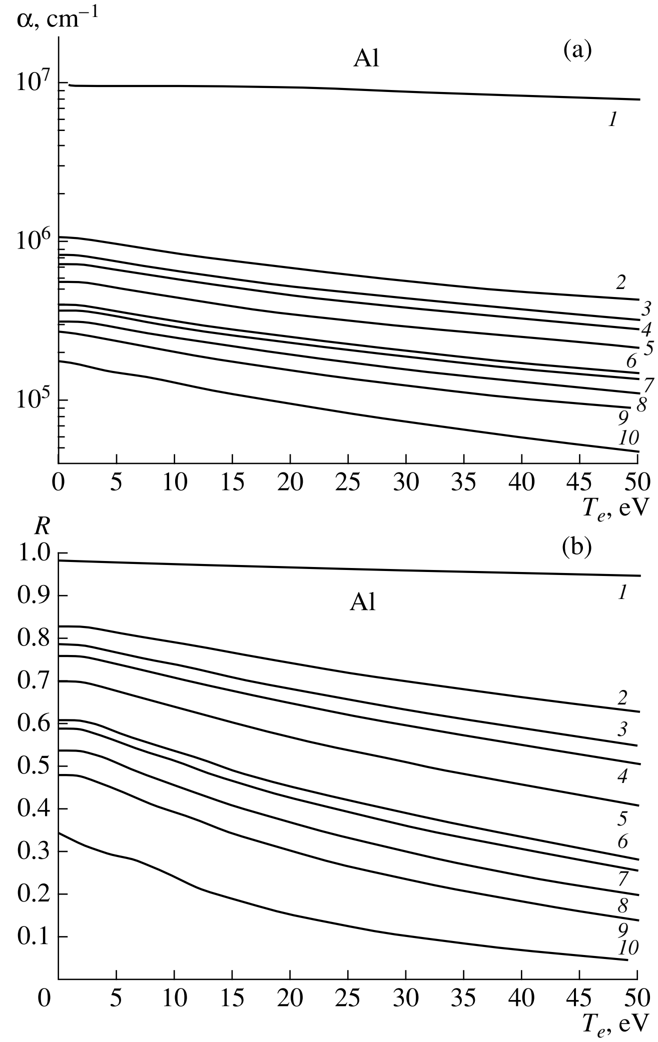

Fig. 3. Temperature dependences (a) volume absorptance (b) surface reflectivity at fixed frequencies: (*1*) $\hbar\omega$ = 0.117 eV (λ = 10.6 μm); (*2*) $\hbar\omega$ = 1.17 eV (λ = 1.06 μm); (*3*) $\hbar\omega$ = 1,55 eV (λ = 0.8 μm); (*4*) $\hbar\omega$ = 1.79 eV (λ = 0.694 μm); (*5*) $\hbar\omega$ = 2.43 eV (λ = 0.51 μm); (*6*) $\hbar\omega$ = 3.68 eV (λ = 0.337 μm); (*7*) $\hbar\omega$ = 4.025 eV (λ = 0.308 μm); (*8*) $\hbar\omega$ = 4.999 eV (λ = 0.248 μm); (*9*) $\hbar\omega$ = 6.42 eV (λ = 0.193 μm); (*10*) $\hbar\omega$ = 12.4 eV (λ = 0.1 μm).

2. CALCULATION OF REFLECTIVITY $R(\Omega, T_E)$ AND VOLUME ABSORPTANCE $\alpha(\Omega, T_E)$ OF ALUMINUM

The results of above theoretical analysis can be used to determine the frequency and temperature dependences of a volume absorptance $\alpha(\omega, T_e)$ and surface reflectivity $R(\omega, T_e)$ of metal targets. Present paper treats aluminum as irradiated target material. Aluminum is a three-valent metal and is characterized by following parameters [14]:

$$N_e = 1.806 \times 10^{23}\ \text{cm}^{-3}, \quad E_F = 11.637\ \text{eV},$$

$$\hbar\omega_{Le} = 15.780\ \text{eV}.$$

As it was already noted, there is extensive information on frequency dependence of optical characteristics of metals [3, 15], measured as a rule at a room temperature. So to compare the calculated and reference data we started with calculation of frequency dependences of imaginary $\varepsilon_2^l(\omega)$ and real $\varepsilon_1^l(\omega)$ parts of permittivity using relationships (20) and (27) at a fixed temperature (293 K), Fig. 1. The results of calculations shown at Fig. 1 indicate that properties of the frequency dependences $\varepsilon_1^l(\omega)$ and $\varepsilon_2^l(\omega)$ correspond to the generally accepted view on the behavior of permittivity of electron plasma. In particular, the real and imaginary parts change with frequency asymptotically tending to 1 and 0 correspondingly, moreover, the real part $\varepsilon_1^l(\omega)$ changes its sign during transition over the point $\hbar\omega = \hbar\omega_{Le}$.

The obtained dependences $\varepsilon_1^l(\omega)$ and $\varepsilon_2^l(\omega)$ were also used to determine the dispersion of the real n and imaginary parts κ of refractivity N and then to calculate the frequency dependences of the volume absorptance $\alpha(\omega)$ and reflectivity of the surface $R(\omega)$ using relations (5) and (6), Fig. 2. Comparison of the obtained dependences $\alpha(\omega)$ and $R(\omega)$ (solid lines) with reference data [3] (dashed lines), Fig. 2, exhibits quite good quantitative fitness in the laser frequency range $\hbar\omega \in [0.1–10.0]$ eV.

The temperature dependences $\alpha(\omega, T_e)$ and $R(\omega, T_e)$ also have been calculated for a number of constant frequencies corresponding to the radiation wavelengths of widely used lasers. Both characteristics, tend to decrease as electron gas temperature rises in the laser range of $\lambda \in (0.1–10.6)$ μm, starting from the infra-red and to the ultraviolet band, Fig. 3.

3. CONCLUSIONS

Ultrashort and high-power pulsed laser treatment of metals is accompanied by generation of strongly nonequilibrium regions in solid with hot electrons and cold lattice. The electron subsystem can be heated to comparable to or exceeding the Fermi energy temperatures. Computational experiments targeting such regimes require determination of optical, thermophysical and other characteristics of irradiated targets in wide temperature and frequency ranges. The present paper suggests an approach to determine the optical characteristics of metals at arbitrary temperature and frequency by an example of calculation of reflectivity and volume absorptance of aluminum surface.

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REFERENCES

1. W. W. Duley, *Laser Processing and Analysis of Materials* (Plenum, New York, 1983).
2. V. N. Zolotaryov, V. I. Morozov, and E. V. Smirnova, *Optical Constants of Natural and Technical Media. Handbook* (Chemistry, Leningrad, 1984) [in Russian].
3. *Handbook of Optical Constants*, Ed. by E. D. Palik (Academic, Boston, San Diego, New York, 1991).
4. *Laser and Electron-Beam Material Processing, Handbook*, Ed. by N. Rykalin, A. Uglov, I. Zuev, and A. Kokora (Mir, Moscow, 1988) [in Russian].
5. J. V. Martynenko and J. N. Javlinsky, Dokl. Akad. Nauk SSSR **270**, 88 (1983).
6. J. M. Ziman, *Principles of the Theory of Solid* (Cambridge Univ., Cambridge, 1964).
7. M. Born and E. Wolf, *Principles of Optics* (Cambridge Univ., Cambridge, 1999; Nauka, Moscow, 1973).
8. *Handbook of Optical Constants*, Ed. by E. D. Palik (Academic, Boston, San Diego, New York, 1991), vol. 2.
9. E. M. Lifshitz and L. P. Pitaevsky, *Physical Kinetics* (Pergamon, Oxford, 1981).
10. V. P. Silin and A. A. Ruhadze, *Electromagnetic Properties of Plasma and Plasma-Like Media* (Gosatomizdat, Moscow, 1961) [in Russian].
11. J. L. Klimontovich and V. P. Silin, Sov. Phys. JETP **23**, 151 (1952).
12. J. V. Martynenko and J. N. Javlinsky, Preprint IAE-3773/1 (Moscow, 1983).
13. J. Lindhard, Det Kongelige Danske Videnskabernes Selskab. Matem.-Fys. Meddelelser **28** (8), 1 (1954).
14. Ch. Kittel, *Introduction to Solid State Physics* (Wiley, New York, 1978).
15. C. W. Allen, *Astrophysical Quantities* (Athlone, London, 1973).