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DYNAMIC ADAPTATION FOR GAS DYNAMICS PROBLEMS

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Summary. A dynamic adaptation method is applied to gas dynamics problems. Using the Woodward–Colella problem and a nonuniformly accelerating piston as examples, the efficiency of the proposed method is demonstrated for the gas dynamics problems with shock wave and contact discontinuity tracking. The grid points are distributed under the control of the diffusion approximation.

1 INTRODUCTION

The main goal of this paper is to further develop the dynamic adaptation method for gas dynamics problems with interacting discontinuous solutions like shock waves, rarefaction waves, and contact discontinuities.

The features of the dynamic adaptation method will be demonstrated using the Woodward-Colella problem, the non-uniformly accelerating piston problem and the modelling piston problem for the gas with non-linear heat conductivity; recently, the former problem has become the most widespread problem for testing new numerical methods in gas dynamics.

The main difficulties are in tracking the discontinuities and in taking into account their interaction. Multiple interactions between discontinuities are characterized by great diversity, but it can be reduced to several elementary interactions: collision of two counter propagating shock waves, absorption of a shock wave by an overtaking shock wave, and passage of a shock wave through a contact discontinuity.

2 MATHEMATICAL STATEMENT OF THE PROBLEMS

The Woodward-Colella problem and the problem of a piston accelerating in an ideal gas is described by the complete system of gas dynamic equations so called Euler equations. According to the dynamic adaptation method (see [1]-[2]), we proceed to an arbitrary nonstationary coordinate system. In the new variables (q, τ), system becomes

$$\frac{\partial}{\partial \tau}\left(\frac{\psi}{\rho}\right)+\frac{\partial}{\partial q}\left(\frac{Q}{\rho}-u\right)=0, \tag{1}$$

$$\frac{\partial}{\partial \tau}(\psi \cdot u) + \frac{\partial}{\partial q}(P + Q \cdot u) = 0, \tag{2}$$

$$\frac{\partial}{\partial \tau}(\psi \cdot \varepsilon) + \frac{\partial}{\partial q}(Q \cdot \varepsilon) + P \cdot \frac{\partial u}{\partial q} = 0 \tag{3}$$

$$\frac{\partial \psi}{\partial \tau} = -\frac{\partial Q}{\partial q}, \tag{4}$$

with the equations of state

$$P = \rho R T, \quad \varepsilon = \frac{R}{\gamma - 1} T$$

Here, ρ is the density, u is the velocity, P is the pressure, ε is the internal energy, T is the temperature, R is the gas constant, γ is the adiabatic index, ψ/ρ is the Jacobian of the inverse transformation and Q is the adaptation function to be determined.

On proceeding to an arbitrary nonstationary coordinate system, the original Euler equations are transformed into extended model (1)–(4), which has been supplemented by the inverse transformation equation (4).

3 THE WOODWARD-COLELLA PROBLEM

The Woodward-Colella problem describes the interaction of two counter propagating detonation waves that are formed as a result of the breakdown of two arbitrary discontinuities in the gas without heat conductivity. From the mathematical point of view, this problem reduces to solving gas-dynamic equations (1) that include the differential conservation laws for the mass, momentum, and energy in the regions $x_l \in [0, 0.1)$, $x_m \in (0.1, 0.9)$, $x_r \in (0.9, 1]$. At the initial moment, the gas with the specific heat ratio $\gamma = 1.4$ is in three different states in these three regions:

$$t = 0: \begin{pmatrix} \rho \\ u \\ P \end{pmatrix}_l = \begin{pmatrix} 1 \\ 0 \\ 10^3 \end{pmatrix}, \begin{pmatrix} \rho \\ u \\ P \end{pmatrix}_m = \begin{pmatrix} 1 \\ 0 \\ 10^{-2} \end{pmatrix}, \begin{pmatrix} \rho \\ u \\ P \end{pmatrix}_r = \begin{pmatrix} 1 \\ 0 \\ 10^2 \end{pmatrix}.$$

At the boundaries $x = 0$ and $x = 1$, the impermeability condition $u(t, 0) = u(t, 1) = 0$ is imposed. At the initial moment, two arbitrary discontinuities are placed at the points $x = 0.1$ and $x = 0.9$.

As a result of the breakdown of discontinuities (6.1) located at the points $x = \{0.1, 0.9\}$, a complex structure is formed in the regions adjacent to these points. This structure includes rarefaction waves, contact discontinuities, and shock waves. In order to determine this structure, the Riemann problem must be solved. At the points of the initial discontinuities are the contact boundaries; from these boundaries, rarefaction waves are propagating towards the outer boundaries of the domain, and shock waves are propagating towards each other. All contact boundaries and shock waves are assumed to be moving and are explicitly tracked. On these discontinuities Hugoniot conditions are satisfied.

The processes in the Woodward-Colella problem were simulated taking into account the abovementioned initial and boundary conditions for system (1)–(4) and the particular form of the adaptation function Q. The main feature of the solution of the problem of interacting counter propagating shock waves is the presence of moving boundaries. The preliminary analysis shows that it is sufficient to use a quasi-uniform grid at each instant of time to solve this problem. A dynamic quasi-uniform distribution of the grid points can be obtained using one of the simplest forms of the function Q that is given by the so-called diffusive approximation [1]:

$$Q = -D \cdot \frac{\partial \psi}{\partial q}, \qquad D = \frac{N \cdot h \cdot \left| Q_l - Q_r \right|}{\psi_{\min}}.$$

Note that the solutions obtained by various methods on the grids with less that 500 points give only a qualitative description of the behavior of gas-dynamic functions; the numerical values can be significantly inaccurate (for example, 10-50% for density). Figure 1 compares the density profiles obtained using the dynamic adaptation method and the modified WENO scheme. It is seen that the dynamic adaptation method on a grid consisting of 420 cells (open dots) produced almost the same solution as WENO5m on 12800 cells (solid line).

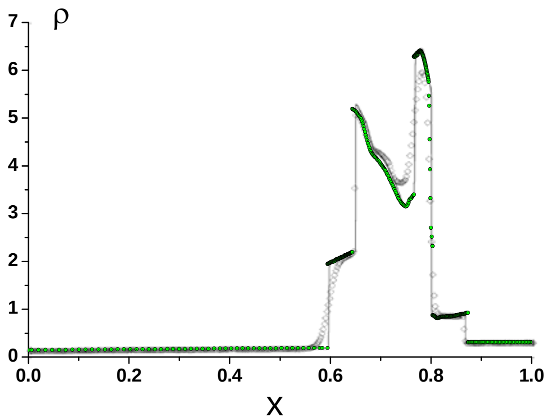

Fig.1.

4 PROBLEM OF A NON-UNIFORMLY ACCELERATING PISTON

The problem concerning a non-uniformly accelerating piston can be obtained by modulating the law of motion of a uniformly accelerating piston by a periodic function of the type $u = V(t) = a_0 \cdot t + V_0 \cdot (1 - \cos(\omega \cdot t))$. The periodic law of motion leads to the generation of a series of shock waves. They appear in the gas near the surface of the piston when it is accelerating and go away into the depth of the domain when it is decelerating. Since the piston law of motion includes an accelerating component, each next shock wave is more intensive than the previous one; when propagating, the next wave overtakes the previous one and absorbs it. At the place of absorption, a contact boundary is formed: a rarefaction wave is formed that propagates towards the piston, and a shock wave propagates in the opposite direction.

Application of the dynamic adaptation method to the non-uniformly accelerating piston problem is reduced to solving system of equations (1)–(4) with the corresponding initial and

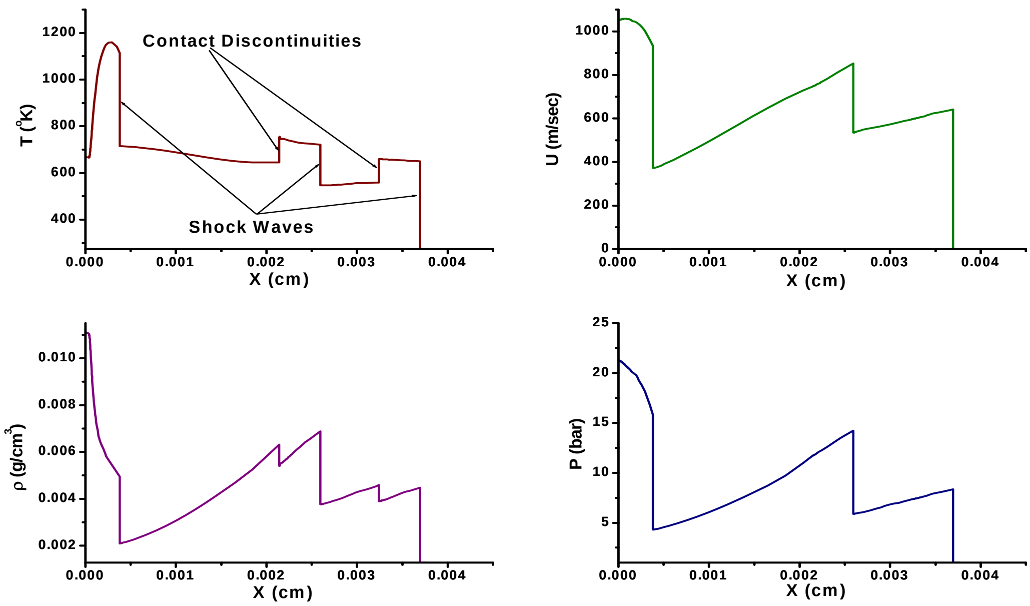

Fig. 2.

boundary conditions [1]. Assuming that the perturbation affects only two intervals (N = 2), formula of the function Q can be modified as follows: $Q = -D \cdot \frac{\partial \psi}{\partial q}$, $D = A(\psi) \cdot \frac{2 \cdot h \cdot |Q_l - Q_r|}{\psi_{\min}}$. The factor $A(\psi) = \begin{cases} 1 & \psi \geq 1 \\ 1/\psi^2 & \psi < 1 \end{cases}$ is introduced to prevent collapsing of two adjacent cells when $\psi < 1$.

The depth of the initial domain and the number of cells were $L = 3.2 \cdot 10^{-4} cm$, $N = 380$. The thermophysical characteristics are as follows: $C_v = 6.93 \cdot 10^{-1}\ J/g \cdot K$ and $R = 2.8668 \cdot 10^{-1}\ J/g \cdot K$. Figure 2 shows the space profiles of the gas-dynamic quantities and temperature at $t = 1.8 \cdot 10^{-7} sec$. Three explicitly tracked shocks and two contact discontinuities are visible; i.e., by this time, the exterior shock was twice absorbed by the shocks formed later. Six space computational domains with one fixed and six explicitly tracked moving boundaries are associated with these phenomena.

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