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Calculation of Optical Properties of Aluminum

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Abstract—In the present work, optical properties of electronic Fermi gas of aluminum are considered at any temperature ($T \geq \varepsilon_F$). The dependences of inductivity are obtained from the solution of the quantum kinetic equation expressions for temperature and frequency. With the help of Fennel's formula, the frequency and temperature dependences of the reflectivity of an irradiated surface and volume factor of absorption are defined.

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INTRODUCTION

Under the influence of concentrated streams of radiation (CSR) on a highly absorbing condensed medium, part of the flow of energy is reflected from the surface and the remainder is absorbed in a thin surface layer. Depending on the mode of exposure (exposure duration, wavelength) of the optical and thermal properties of processed materials, the selection of the energy allocation has a superficial or volumetric character. The emitted energy is further expended for heating, melting, and evaporation of the target.

A systematic study of all phenomena that have developed in the impact zone of the CSR is necessary for the formulation of the basic requirements for sources of CSR and determination of the optimum exposure. Thus, in the laser processing of opaque materials, it is necessary to know optical characteristics such as surface reflectivity R [%] and related absorptivity $A = 1 - R$, and if the energy release is voluminous, then it is necessary to know also the volume absorption coefficient α cm^{-1}. For a broad class of materials, including metals, there is fairly comprehensive information on the frequency dependence of the optical characteristics [1–3] measured at a fixed temperature, usually room temperature. The temperature dependences for most materials, even at low temperatures, are not determined exactly. Thus, for most metals at temperatures below the melting temperature, it is well known that the temperature dependence of the absorbance has a linear character $A(T) = a + bT$, where a and b are coefficients. For temperatures near the melting point of the substance or exceeding it, we believe that the reflectivity/absorptivity does not depend on temperature, and estimates of its average or its value mean integral are used over the required range of temperatures [4].

Such an approach is not useful for mathematical modeling of the impact on metals and super (power) ultrashort laser pulses. As a result of such influences, the radiation energy is transferred to electrons and in solids a strongly nonequilibrium region is formed with hot electrons and a cold lattice. Thus, for small times ($t \sim 10^{-12}$–10^{-15} s), the warming of the electronic subsystem to the temperature T_e comparable to the Fermi energy E_F or exceeding it is possible. In the case of $T_e < E_F$, in a hot field, the electrons have the Fermi distribution with its temperature. When $T_e \cong E^F$, degeneracy is lifted, and for $T_e > E_F$, the electrons acquire a Maxwellian distribution. Transition through the temperature $T_e \sim E_F$ is associated with changes in the mechanisms of electron-electron and electron-phonon interactions, which leads to qualitative changes in the optical and thermal characteristics of a solid [5].

In this paper we attempt to calculate the temperature and frequency dependences of the optical characteristics of metals in a wide range of frequencies ($\hbar\omega = 0.1$–10 eV) and temperatures ($T_e = 0.024$–50 eV). To achieve the goal, we used the longitudinal dielectric constant $\varepsilon^l = \varepsilon^l(\omega, T)$, determined by solving a quantum kinetic equation.

The main limitation of the proposed approach is the description of processes in the model of free gas. The electronic excitation of ions and an increase in the number of free electrons, typical for temperatures $T_e > E_F$ and photoprocesses characteristic for high irradiance, are not taken into account in this review.

I. THEORETICAL ANALYSIS

1. Reflectivity and the Volume Absorption Coefficient

All linear (macroscopic) optical properties of plasma, including the material absorption coefficient, reflectivity R, and the complex refractive index $N = n + i\kappa$, can be expressed by its dielectric constant [6]. By definition, the complex refractive index N is equal to the value

$$N = n + i\kappa = \sqrt{\varepsilon^l}, \tag{1}$$

where n and κ are optical constants that represent the real and imaginary parts of the refractive index, and ε^l is the longitudinal dielectric constant. Given the fact that the dielectric constant is also a complex value, $\varepsilon^l = \varepsilon_1^l + i\varepsilon_2 l$, then, by equating their real and imaginary parts, we obtain the system of equations

$$\varepsilon_1^l = n^2 - \kappa^2, \quad \varepsilon_2^l = 2n\kappa. \tag{2}$$

From the solution of system (2), the expressions for n and κ follow:

$$n = 2^{-1/2}\{\varepsilon_1^l + [(\varepsilon_1^l)^2 + (\varepsilon_2^l)^2]^{1/2}\}^{1/2}, \tag{3}$$

$$\kappa = 2^{-1/2}\{-\varepsilon_1^l + [(\varepsilon_1^l)^2 + (\varepsilon_2^l)^2]^{1/2}\}^{1/2}. \tag{4}$$

According to the classical Fresnel formulas [7], the reflectivity R and absorption A capacity of the irradiated surface of an infinitely thick plasma layer at normal incidence is expressed as

$$R = \frac{(n-1)^n + \kappa^2}{(n+1)^2 + \kappa^2}, \quad A = 1 - R = \frac{4n}{(n+1) + \kappa^2}. \tag{5}$$

The coefficient of volume absorption α is equal to [6]

$$\alpha = \frac{2\kappa\omega}{c} = \frac{4\pi\kappa}{\lambda}, \tag{6}$$

where ω is the frequency of radiation, λ is the wavelength, and c is the speed of light.

Usually, at low temperatures, the values R, A, u, and α are determined by the measurement data, from which the optical constants n and κ are found [6–8].

For the electron plasma of metals, in the general case, n, κ, and ε^l are functions of the radiation frequency ω and temperature T

$$n = n(\omega, T), \quad \kappa = \kappa(\omega, T), \quad \varepsilon^l = \varepsilon^l(\omega, T).$$

Thus, all the macroscopic optical properties of metals and their frequency and temperature dependence can be expressed through a single quantity, the longitudinal dielectric constant $\varepsilon^l = \varepsilon^l(\omega, T)$.

2. The Permittivity of a Degenerate Electron Plasma Metal $\varepsilon(\omega, \kappa)$

According to the electromagnetic field theory, the permittivity of free electron gas $\varepsilon(\omega, \mathbf{k})$ depends on the frequency ω (the so-called time or frequency dispersion) and wave vector $\mathbf{k}$ (spatial dispersion). In the presence of spatial dispersion, i.e., in the presence of the dependence of vector $\mathbf{k}$, permittivity is a tensor quantity $\varepsilon_{ij}(\omega, \mathbf{k})$ even in an isotropic medium. Tensor $\varepsilon_{ij}(\omega, \mathbf{k})$ is characterized by two scalar functions, ε^l and ε^t, dubbed the longitudinal and transverse permittivities, respectively, and that depend on the independent variables, frequency ω and the absolute magnitude of the wave vector k, $\varepsilon^l = \varepsilon^l(\omega, k)$, $\varepsilon^t = \varepsilon^t(\omega, k)$. For $\mathbf{k} \longrightarrow 0$, the selected direction vanishes and the tensor $\varepsilon_{ij}(\omega, \mathbf{k})$ reduces to the form $\varepsilon(\omega)\delta_{ij}$, where $\varepsilon(\omega)$ is the usual scalar permeability, taking into account only the frequency dispersion. Accordingly, the limiting values of the functions ε^l, ε^{tr} are the same and equal $\varepsilon^l = \varepsilon^l(\omega, 0) = \varepsilon^t = \varepsilon^t(\omega, 0) = \varepsilon(\omega)$.

Tensor $\varepsilon_{ij}(\omega, \mathbf{k})$, in the general case, is a complex function of real variables ω and $\mathbf{k}$. The scalar functions ε^l and ε^t are also complex functions of frequency and magnitude of the wave vector k

$$\varepsilon^l(\omega, k) = \varepsilon_1^l(\omega, k) + i\varepsilon_2^l(\omega, k), \quad \varepsilon^t(\omega, k) = \varepsilon_1^t(\omega, k) + i\varepsilon_2^t(\omega, k). \tag{7}$$

The presence of temporal and spatial dispersion in the longitudinal permittivity $\varepsilon^l(\omega, k)$ allows us to determine its frequency and temperature dependences, respectively, as well as the similar dependences of the refractive indices and absorption.

When determining the longitudinal permittivity in an arbitrary temperature range, it was assumed that the transition from the case of strong degeneracy $\xi = T_e/E_F \ll 1$ to the Boltzmann degeneracy $\xi = T_e/E_F \gg 1$ occurs smoothly. For the smooth transition, the temperature dependence of chemical potential was used [6]:

$$\mu(T_e) \cong E_F\left(1 - \frac{\pi^2}{3!}\xi^2 + \frac{\pi^4}{3!4!}\xi^4\right) = E_F D(\xi),$$
$$D(\xi) = \left(1 - \frac{\pi^2}{3!}\xi^2 + \frac{\pi^4}{3!4!}\xi^4\right). \tag{8}$$

At low temperatures, when the value of $\mu(T_e)$ is close to E_F (exact equality $\mu(T_e) = E_F$ is achieved at absolute zero), the collectivized electrons of the metal obey the Fermi-Dirac statistics $f(E) = (\exp(x - \eta) + 1)^{-1}$, where $x = E/T_e$, $\eta = \mu(T_e)/T_e$, and E is the energy of an electron. The scope of the distribution function $f(E)$, corresponding to large values of energy E, when $x - \eta \gg 1$ is essentially similar to the Boltzmann distribution function $f(E) = \exp(\eta - x)$.

The value of the total longitudinal permittivity is proposed to determine the sum of two components, ensuring a smooth transition from a degenerate electron gas to the Maxwellian plasma

$$\varepsilon^l = \varepsilon^l(\omega, T) = D(\xi)\varepsilon_F^l(\omega, T) + (1 - D(\xi))\varepsilon_M^l(\omega, T). \tag{9}$$

3. Quantum Kinetic Equation

In the general case of arbitrary values of vector $\mathbf{k}$, when a significant role is played by spatial dispersion, the calculation of the permittivity $\varepsilon(\omega, \mathbf{k})$ requires the application of kinetic equation [9], which for a collisionless plasma has the form

$$\frac{\partial(\delta f)}{\partial t} + \mathbf{v}\frac{\partial(\delta f)}{\partial \mathbf{r}} = e\mathbf{E}\frac{\partial(\delta f_0)}{\partial \mathbf{p}}, \tag{10}$$

where $f(\mathbf{p}) = f_0 + \delta f(p)$ is the electron distribution function in momentum space, f_0 is the stationary isotropic and spatially uniform distribution function that is unperturbed by the field, and δf is the change of the distribution function under the influence of the field.

The longitudinal part of the permittivity $\varepsilon^l(\omega, k)$ of collisionless plasma is determined by solving kinetic Eq. (10) and has the form [9]

$$\varepsilon^l(\omega, k) = 1 - \frac{4\pi e^2}{k^2}\int_{-\infty}^{\infty}\mathbf{k}\frac{\partial f(\mathbf{p})}{\partial \mathbf{p}}\frac{d^3p}{\mathbf{kv} - \omega - i0}. \tag{11}$$

This expression has a singular point (pole) in the lower half of the complex variable. Peculiarity $\frac{1}{\mathbf{kv} - \omega}$ is usually [9, 10] considered as $\frac{1}{\mathbf{kv} - (\omega + i0)}$; i.e., the value ω is represented as the value $\omega + i0$, which has an infinitesimal positive imaginary part. After renaming the variable of integration, integral (11) can be written as

$$\int_{-\infty}^{\infty}\frac{f(z)}{z - i\delta}dz, \quad \delta > 0.$$

In this integral the path of integration in the complex plane z passes under the point $z = i\delta$, which at $\delta \longrightarrow 0$ is equivalent to integration along the real axis around the pole $z = 0$ along an infinitesimal semicircle below. The contribution to the integral of this bypass is determined by the semiresidue of integrand

$$\int_{-\infty}^{\infty}\frac{f(z)}{z - i\delta}dz = \int_{-\infty}^{\infty}\frac{f(z)}{z}dz + i\pi f(0). \tag{12}$$

The integral on the right side of Eq. (12) represents the limiting value of the Cauchy type integral.

To describe the electromagnetic properties of the quantum plasma, to which the electron plasma of metals is related, the quantum analogue of the classical kinetic equation is used [10]. The equation for the quantum distribution function $f(\mathbf{p})$, depending on the kinematic momentum $\mathbf{p}$, for a small deviation from the equilibrium homogeneous state $f(\mathbf{p}, \mathbf{r}, t) = f_0(\mathbf{p}) + \delta f(\mathbf{p}, \mathbf{r}, t)$[10] has the form

$$\frac{\partial(\delta f)}{\partial t} + \mathbf{v}\frac{\partial(\delta f)}{\partial \mathbf{r}} + e\mathbf{E}\frac{\partial(\delta f_0)}{\partial \mathbf{p}} = \frac{e}{(2\pi)^3}\frac{i}{\hbar}\int e^{i\tau(p'-p)} f_0(\mathbf{p}')\left\{\left[\frac{\partial\varphi}{\partial \mathbf{r}}\hbar\boldsymbol{\tau} - \varphi\left(\mathbf{r} + \frac{\hbar\boldsymbol{\tau}}{2}\right)\right.\right.$$
$$\left.\left. + \varphi\left(\mathbf{r} - \frac{\hbar\boldsymbol{\tau}}{2}\right)\right] - \frac{\mathbf{v}'}{c}\left[\left(\hbar\boldsymbol{\tau}\frac{\partial \mathbf{A}}{\partial \mathbf{r}}\right) - \mathbf{A}\left(\mathbf{r} + \frac{\hbar\boldsymbol{\tau}}{2}\right) + \mathbf{A}\left(\mathbf{r} - \frac{\hbar\boldsymbol{\tau}}{2}\right)\right]\right\} d\boldsymbol{\tau}\, d\mathbf{p}'. \tag{13}$$

where f_0 is the stationary isotropic and spatially homogeneous electron distribution function in momentum that is unperturbed by the field and δf is its change under the influence of the field; $\mathbf{E}$, φ, and $\mathbf{A}$ are the electric field intensity vector, the scalar potential, and the vector potential, respectively; and $\boldsymbol{\tau} = \mathbf{p}^{-1}$.

In the classical limit $\hbar \longrightarrow 0$, Eq. (13) transforms into kinetic Eq. (10).

For a degenerate electron gas function, $f(\mathbf{p})$ is the Fermi distribution function, equal to

$$f_F(\mathbf{p}) = \frac{2n(\mathbf{p})}{(2\pi\hbar)^3},$$

where $\frac{2d^3p}{(2\pi\hbar)^3}$ is the number of states attributable to an element of momentum space d^3p with two values of spin projections, and $n(\mathbf{p})$ is the number of filled quantum states of electrons with specific values of momentum and spin projections. In the case of complete degeneracy ($T = 0$) and $n(\mathbf{p}) = 1$, and the distribution function takes the form

$$f_F(\mathbf{p}) = \begin{cases} \dfrac{2}{(2\pi\hbar)^3}, & p < p_F = (3\pi^2)^{1/3}\hbar N_e^{1/3}, \\ 0, & p > p_F. \end{cases} \tag{14}$$

The formula for the longitudinal permittivity $\varepsilon^l(\omega, \mathbf{k})$ of a completely degenerate electron gas with distribution function (14) was obtained in [11] from the solution of quantum kinetic Eq. (13):

$$\varepsilon^l(\omega, \mathbf{k}) = 1 - \frac{4\pi e^2}{\hbar k^2}\int \frac{\left[n\left(\mathbf{p} + \frac{\hbar\mathbf{k}}{2}\right) - n\left(\mathbf{p} - \frac{\hbar\mathbf{k}}{2}\right)\right]}{\mathbf{k}\upsilon - (\omega + i0)}\frac{2d^3p}{(2\pi\hbar)^3}. \tag{15}$$

A more general expression for the dielectric constant of the degenerate plasma is obtained at $T \neq 0$. The simple, though cumbersome, integration of Eq. (15) leads to [12]

$$\varepsilon^l(\omega, k) = 1 - \frac{4\pi m e^2 N_e}{\hbar k^3 p_{Te} F_{1/2}(\xi)}\int_{-\infty}^{\infty} \ln\left[\frac{1 + \exp\left(\eta(\xi) - \left(\frac{p}{p_{Te}} - \frac{\hbar k}{2p_{Te}}\right)^2\right)}{1 + \exp\left(\eta(\xi) - \left(\frac{p}{p_{Te}} + \frac{\hbar k}{2p_{Te}}\right)^2\right)}\right]\frac{dp}{kp/m - (\omega + i0)}, \tag{16}$$

where ϑ_{Te} and p_{Te} are, respectively, the average thermal velocity and momentum of the electrons and k is the average absolute value of wave vector $\mathbf{k}$.

Because of the presence of peculiarities $\frac{1}{\mathbf{k}\upsilon - (\omega + i0)}$, the integral in Eq. (16) represents a limiting value of the Cauchy type integral. Integration is performed in the complex plane $(\omega + i0)$ along the real axis, with a bypass of point $p = m\omega/k$ from below.

4. The Imaginary Part of the Longitudinal Permittivity $\varepsilon_2^l(\omega, \mathrm{k})$

The imaginary part of $\varepsilon_2^l(\omega, \mathbf{k})$ of the longitudinal permittivity $\varepsilon^l(\omega, \mathbf{k})$ is defined in (16) by the semi-residue at the point $p = m\omega/k$. Separating it with the help of formula (12), we obtain

$$\varepsilon_2^l(\omega, k) = \frac{4\pi^2 e^2 m N_e}{\hbar k^3 p_T F_{1/2}(\xi)} \ln\left\{\frac{1 + \exp\left(\eta(\xi) - \left(\frac{p}{p_{Te}} - \frac{\hbar k}{2p_{Te}}\right)^2\right)}{1 + \exp\left(\eta(\xi) - \left(\frac{p}{p_{Te}} + \frac{\hbar k}{2p_{Te}}\right)^2\right)}\right\}. \quad (17)$$

The imaginary part of permittivity $\varepsilon_{2,F}^l(\omega, k)$ in the case of a degenerate electron Fermi gas, taking into account the expressions for the main variables $p = m\omega/k$, $k_F = p_F/\hbar$, $p_F = m\upsilon_F = (2m\varepsilon_F)^{1/2}$, $\upsilon_F = (2\varepsilon_F/m)^{1/2}$, has the form

$$\varepsilon_{2,F}^l(\omega, k) = 3\pi \frac{T_e(\hbar\omega_{Le})^2}{(\hbar\omega)^3} \ln\left\{\frac{1 + \exp\left(\eta(\xi) - \left(\frac{\omega}{k_F\upsilon_{Te}} - \frac{\hbar k_F}{2m\upsilon_{Te}}\right)^2\right)}{1 + \exp\left(\eta(\xi) - \left(\frac{\omega}{k_F\upsilon_{Te}} + \frac{\hbar k_F}{2m\upsilon_{Te}}\right)^2\right)}\right\}, \quad (18)$$

where $\omega_{Le} = \dfrac{\upsilon_{Te}}{d_e} = \left(\dfrac{4\pi e^2 N_e}{m}\right)^{1/2}$, $d_e = \left(\dfrac{T_e}{4\pi e^2 N_e}\right)$ is the plasma, or Langmuir, frequency and Debye radius for electrons, respectively, and $F_{k+1/2} = \int_0^\infty \dfrac{x^{k+1/2}}{\exp(x - \eta) + 1}\, dx$ is the Fermi integral. Integral $F_{1/2}$ and the value $\eta(\xi)$ are approximated by the expressions [5]

$$F_{1/2} \cong \frac{2}{3}\xi^{-3/2}, \quad \eta(\xi) = \xi^{-1} + \ln\frac{4/3\pi^{1/2}}{\xi^{3/2} + 4/3\pi^{1/2}}.$$

For a Maxwellian plasma, taking into account $k_{Te} = p_{Te}/\hbar = \hbar^{-1}(2mT_e)^{1/2}$, $p_{Te} = m\upsilon_{Te} = (2mT_e)^{1/2}$, $\vartheta_{Te} = (2T_e/m)^{1/2}$, the imaginary part of permittivity $\varepsilon_{2,M}^l(\omega, k)$ has the form

$$\varepsilon_{2,M}^l(\omega, k) = 2\pi \frac{T_e(\hbar\omega_{Le})^2}{(\hbar\omega)^3} \ln\left\{\frac{1 + \exp\left(\eta(\xi) - \left(\frac{\omega}{k_{Te}\upsilon_{Te}} - \frac{\hbar k_{Te}}{2m\upsilon_{Te}}\right)^2\right)}{1 + \exp\left(\eta(\xi) - \left(\frac{\omega}{k_{Te}\upsilon_{Te}} + \frac{\hbar k_{Te}}{2m\upsilon_{Te}}\right)^2\right)}\right\}. \quad (19)$$

In the two limiting cases of low $T_e \longrightarrow 0$ and high $T_e \gg \varepsilon_F$ temperatures, expressions (18) and (19) transit to two well-known relations. At $T_e \longrightarrow 0$, expression (18) transits to the formula obtained in [13] for the imaginary part of permittivity of $\varepsilon_2^l(\omega, k)$ of the degenerate electron Fermi gas:

$$\varepsilon_{2,F}^l(\omega, k) = \frac{3\pi}{2}\frac{\omega_{Le}^2\omega}{(k\upsilon_F)^3} \quad \text{at} \quad |\omega| < k\vartheta_F.$$

In the other limiting case of high temperatures $\xi \gg 1$, it is sufficient to remove the degeneracy, and $h \longrightarrow 0$; relation (19) coincides with the well-known expression for $\varepsilon_{2,M}^l(\omega, k)$ of the classical electron plasma [9]:

$$\varepsilon_2^l(\omega, k) = \left(\frac{\pi}{2}\right)^{1/2}\frac{\omega\omega_{Le}^2}{(k\upsilon_{Te})^3}\exp\left(-\frac{1}{2}\left(\frac{\omega}{k\upsilon_{Te}}\right)^2\right).$$

Finally, the imaginary part of permittivity of the degenerate electron gas $\varepsilon_{2,F}^{l}(\omega, k)$ and Maxwell plasma $\varepsilon_{2,M}^{l}(\omega, k)$ can be represented as a frequency and temperature dependence

$$\varepsilon_{2,F}^{l}(\omega, T_e) = 3\pi T_e \frac{(\hbar\omega_{Le})^2}{(\hbar\omega)^3} \ln\left\{\frac{1+\exp\left(\eta(\xi)-\frac{E_F}{4T_e}\left(\frac{\hbar\omega}{E_F}-1\right)^2\right)}{1+\exp\left(\eta(\xi)-\frac{E_F}{4T_e}\left(\frac{\hbar\omega}{E_F}+1\right)^2\right)}\right\},$$

$$\varepsilon_{2,M}^{l}(\omega, T_e) = 2\pi T_e \frac{(\hbar\omega_{Le})^2}{(\hbar\omega)^3} \ln\left\{\frac{1+\exp\left(\eta(\xi)-\frac{1}{4}\left(\frac{\hbar\omega}{T_e}-1\right)^2\right)}{1+\exp\left(\eta(\xi)-\frac{1}{4}\left(\frac{\hbar\omega}{T_e}+1\right)^2\right)}\right\}.$$

Finally, the expression for the total imaginary part of permittivity ε_2^l according to (9) is represented as a ratio explicitly depending on the radiation frequency ω and the electron temperature T_e:

$$\varepsilon_2^l(\omega, T_e) = D(\xi)\varepsilon_{2,F}^{l}(\omega, T_e) + (1-D(\xi))\varepsilon_{2,M}^{l}(\omega, T_e) = \left(C\frac{T_e(\hbar\omega_{Le})^2}{(\hbar\omega)^3}\right)$$

$$\times\left\{D(\xi)\ln\left[\frac{1+\exp\left(\eta(\xi)-\frac{1}{4\xi}\left(\frac{\hbar\omega}{E_F}-1\right)^2\right)}{1+\exp\left(\eta(\xi)-\frac{1}{4\xi}\left(\frac{\hbar\omega}{E_F}+1\right)^2\right)}\right] + (1-D(\xi))\ln\left[\frac{1+\exp\left(\eta(\xi)-\frac{1}{4\xi}\left(\frac{\hbar\omega}{T_e}-1\right)^2\right)}{1+\exp\left(\eta(\xi)-\frac{1}{4\xi}\left(\frac{\hbar\omega}{T_e}+1\right)^2\right)}\right]\right\}, \quad (20)$$

where $D(\xi) = \left(1-\frac{\pi^2}{3!}\xi^2+\frac{\pi^4}{3!4!}\xi^4\right)$, $C = \mathrm{C}(\hbar\omega)$ is a value that is slowly varying by frequency, and $C = 1-2\pi$ in the frequency range $\hbar\omega = 0.1-100$ eV.

5. The Real Part of The Longitudinal Permittivity $\varepsilon_1^l(\omega, \mathbf{k})$

Determination of the real part of permittivity $\varepsilon_1^l = \varepsilon_1^l(\omega, \mathbf{k})$ using Eq. (16) analytically is possible only for two limiting cases [12]: for high $\omega/k\upsilon_{Te} \gg 1$ and low-frequency $\omega/k\upsilon_{Te} \ll 1$.

5.1. High-frequency approximation of $\varepsilon_1^{l,h}(\omega, k)$. At high frequencies the $\omega/k\upsilon_{Te} \gg 1$ integrand of (16) can be expanded in powers of k/ω and after integration obtain the approximation to the real part of permittivity $\varepsilon_1^{l,h}(\omega, k)$:

$$\varepsilon_1^{l,h}(\omega, k) = 1 - \frac{4\pi e^2 N_e}{kp_{Te}} \frac{1}{\omega F_{1/2}} \sum_{j=0}^{\infty}\left[\left(\frac{k\upsilon_{Te}}{\omega}\right)^{2j+1} F_{(2j+1)/2}\right]. \quad (21)$$

Retaining the first three terms in the expansion in (21), we obtain

$$\varepsilon_1^{l,h}(\omega, T_e) = 1 - \frac{\omega_{Le}^2}{k\upsilon_{Te}\omega}\left(\left(\frac{k\upsilon_{Te}}{\omega}\right) + \left(\frac{k\upsilon_{Te}}{\omega}\right)^2 \frac{F_{3/2}}{F_{1/2}} + \left(\frac{k\upsilon_{Te}}{\omega}\right)^5 \frac{F_{5/2}}{F_{1/2}} + \ldots\right).$$

For the Fermi component

$$\varepsilon_{1,F}^{ih}(\omega, T_e) = 1 - \left(\frac{\omega_{Le}}{\omega}\right)^2\left(1 + \left(\frac{k_F\upsilon_{Te}}{\omega}\right)^2 \frac{F_{3/2}}{F_{1/2}} + \left(\frac{k_F\upsilon_{Te}}{\omega}\right)^4 \frac{F_{5/2}}{F_{1/2}} + \ldots\right).$$

Taking into account the values of the Fermi integrals $F_{k+1/2}$ and the average electron energy $\langle E\rangle = T_e\frac{F_{3/2}}{F_{1/2}}$, the approximation for $\varepsilon_{1,F}^{l,h}$ is written as

$$\varepsilon_{1,F}^{l,h} \approx 1 - \left(\frac{\hbar\omega_{Le}}{\hbar\omega}\right)^2\left[1 + \frac{4E_F\langle E\rangle}{(\hbar\omega)^2} + \frac{(4E_F)^2\langle E\rangle^2}{(\hbar\omega)^4}\right]. \tag{22}$$

For Maxwellian plasma $\xi > 1$

$$\begin{aligned}\varepsilon_{1,M}^{l,h}(\omega, T_e) &= 1 - \left(\frac{\hbar\omega_{Le}}{\hbar\omega}\right)^2\left(1 + \left(\frac{k_{T_e}\upsilon_{Te}}{\omega}\right)^2 + \left(\frac{k_{T_e}\upsilon_{Te}}{\omega}\right)^4\right)\\ &= 1 - \left(\frac{\hbar\omega_{Le}}{\hbar\omega}\right)^2\left(1 + \left(\frac{2T_e^2}{\hbar\omega}\right)^2 + \left(\frac{2T_e^2}{\hbar\omega}\right)^4\right).\end{aligned} \tag{23}$$

5.2 Low-frequency approximation $\varepsilon_1^{l,s}$. For low frequencies $\omega/k\upsilon_T \ll 1$ in integral (16), it is necessary to change the variables $y = x + \omega/k\upsilon_T$, expand the integrand into a series of $\omega/k\upsilon_{Te}$, and after integration over x to obtain

$$\varepsilon_1^{l,s} = 1 - \frac{4\pi e^2 N_e}{k^2 p_{Te}^2}\frac{m}{F_{1/2}}\left(F_{-1/2} + \sum_{j=0}^{\infty}(-1)^j\frac{2^{2j+1}j!}{(2j+1)!}\frac{2+j}{1+j}\left(\frac{\omega}{k\upsilon_T}\right)^{2(j+1)}\int_0^{\infty}\frac{f^{(j)}(x)}{\sqrt{x}}dx\right), \tag{24}$$

where $f^{(j)}(x)$ is the jth derivative of the Fermi distribution function. Keeping the first two terms of expansion $j = 0, 1$, in (24), we write

$$\varepsilon_1^{l,s}(\omega, k) = 1 - \frac{4\pi e^2 N_e}{k^2 p_{Te}^2}\frac{m}{F_{1/2}}\left(F_{-1/2} + 4\left(\frac{\omega}{k\upsilon_T}\right)^2\int_0^{\infty}\frac{f^{(0)}(x)}{\sqrt{x}}dx - 2\left(\frac{\omega}{k\upsilon_T}\right)^4\int_0^{\infty}\frac{f^{(1)}(x)}{\sqrt{x}}dx\right).$$

For $\xi < 1$, for the Fermi distribution we obtain

$$\varepsilon_{1,F}^{l,s}(\omega, k) \approx 1 - \frac{4\pi e^2 N_e}{k_F^2 p_{T_e}^2}\frac{F_{-1/2}}{F_{1/2}}\left[1 + 4\left(\frac{\omega}{k_F\upsilon_T}\right)^2 + 2\left(\frac{\omega}{k_F\upsilon_T}\right)^4\right].$$

Taking into account $\int_0^{\infty}\frac{f^{(0)}(x)}{\sqrt{x}}dx = \int_0^{\infty}\frac{x^{-1/2}dx}{\exp(x-\eta)+1} = F_{-1/2}$ and $\frac{F_{-1/2}}{F_{1/2}} = \frac{2\xi}{(\xi^2 + (2/3)^2)^{1/2}}$, the low-frequency approximation for the Fermi component takes the form

$$\varepsilon_{1,F}^{l,s}(\omega, k) \approx 1 - \frac{(\hbar\omega_{Le})^2}{E_F^2}\frac{2}{(\xi^2 + (4/9))^{1/2}}\left(1 + \frac{(\hbar\omega)^2}{E_F T_e} + \frac{1}{8}\frac{(\hbar\omega)^4}{(E_F T_e)^2}\right). \tag{25}$$

For the Maxwellian component $\xi > 1$, the corresponding component has the form

$$\varepsilon_{1,M}^{l,s}(\omega, k) \approx 1 - \frac{1}{2}\left(\frac{\hbar\omega_{Le}}{T_e}\right)^2\left[1 + \left(\frac{\hbar\omega}{T_e}\right)^2 + \frac{1}{8}\left(\frac{\hbar\omega}{T_e}\right)^4\right]. \tag{26}$$

Taking into account expressions (22) and (25), the real part of permittivity for a degenerate electron gas $\varepsilon_{1,F}^{l}$ can be represented in the form of matching the high-frequency $\varepsilon_{1,F}^{l,h}(\omega, T_e)$ and low-frequency $\varepsilon_{1,F}^{l,s}(\omega, T_e)$ approximations.

The high-frequency approximation $\varepsilon_{1,F}^{l,h}(\omega, T_e)$, satisfying the condition $\omega/k\upsilon_{Te} \gg 1$, is used at low temperatures, and the low-frequency $\varepsilon_{1,F}^{l,s}(\omega, T_e)$, satisfying the condition $\omega/k\upsilon_{Te} \ll 1$ is used for high temperatures. Stitching is carried out at the point of the intersection of curves $\varepsilon_{1,F}^{l,h}(\omega, T_e)$ and $\varepsilon_{1,F}^{l,s}(\omega, T_e)$. At

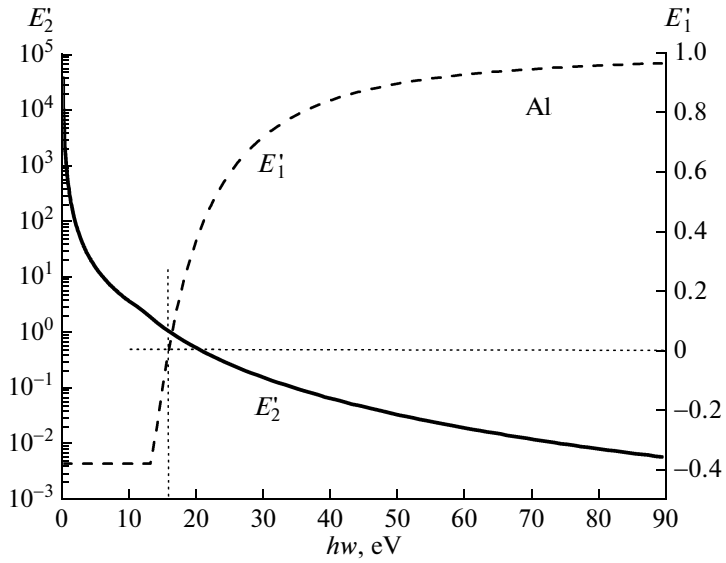

Fig. 1. Frequency dependence of the real and imaginary parts of $E_1^l(\omega)$ of the longitudinal permittivity at a temperature of $T = 290$ K.

this point, at the movement on the temperature scale a transition from the curve $\varepsilon_{1,F}^{l,h}(\omega, T_e)$ to the curve $\varepsilon_{1,F}^{l,s}(\omega, T_e)$ occurs.

Similarly, the high-frequency $\varepsilon_{1,M}^{l,h}(\omega, T_e)$ and low-frequency $\varepsilon_{1,M}^{l,s}(\omega, T_e)$ approximation of Maxwell's plasma are merged and the real part of permittivity $\varepsilon_{1,M}^{l}(\omega, T_e)$ is determined.

Using these expressions for $\varepsilon_{1,F}^{l}$ and $\varepsilon_{1,M}^{l}(\omega, T_e)$, the real part of permittivity $\varepsilon_1^{l,h}(\omega, k)$ in its final form can be represented as

$$\varepsilon_1^l(\omega, T) = D(\xi)\varepsilon_{1,F}^l(\omega, T) + (1 - D(\xi))\varepsilon_{1,M}^l(\omega, T). \quad (27)$$

II. CALCULATION OF THE REFLECTIVITY $R(\Omega, T_e)$ AND THE VOLUME ABSORPTION COEFFICIENT $A(\Omega, T_e)$ OF ALUMINUM

The results of the above theoretical analysis can be used to determine the frequency and the temperature dependences of the volume absorption coefficient $\alpha(\omega, T_e)$ and surface reflectance $R(\omega, T_e)$ of metal targets. In the present work, aluminum, which is a trivalent metal and is characterized by the following parameters, was considered as a metal target [14]:

$$N_e = z\frac{\rho}{AM} = 1.806 \times 10^{23}\ \text{cm}^{-3}, \quad E_F = \frac{\hbar^2}{2m}(3\pi^2 N_e)^{2/3} = 11.637\ \text{eV},$$

$$\hbar\omega_{Le} = (4\pi e^2 N_e)^{1/2} = 15.780\ \text{eV},$$

where z is valence, A is atomic weight, and M is the atomic mass unit.

As already noted, there is extensive information in the literature on the frequency dependence of the optical properties of metals [2, 3, 15], measured usually at room temperature. Thus, at first, in order to compare the calculated data with the background data using relations (20) and (27), the frequency dependences on the imaginary $\varepsilon_2^l(\omega)$ and the real $\varepsilon_1^l(\omega)$ parts of the permittivity were calculated at a fixed temperature ($T_e = 293$ K) (see Fig. 1). The calculations shown in Fig. 1 indicate that the features of the frequency dependences $\varepsilon_1^l(\omega)$ and $\varepsilon_2^l(\omega)$ comply with generally accepted notions about the behavior of the permittivity of the electron plasma. Thus, the real and imaginary parts of the frequency increase, showing an asymptotic tendency to unity and zero, respectively; see Fig. 1. In addition, when passing through the

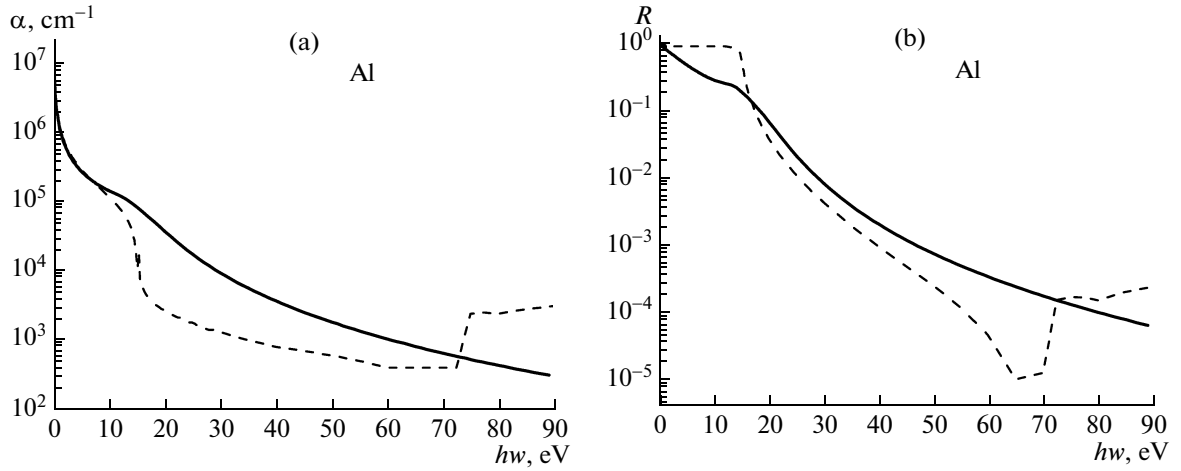

Fig. 2. Frequency dependence of (a) the volume absorption coefficient $\alpha(\omega)$, and (b) the surface reflectivity coefficient $R(\omega)$ at a temperature of $T = 290$ K. The solid lines are the calculated curves and dotted lines are the reference data.

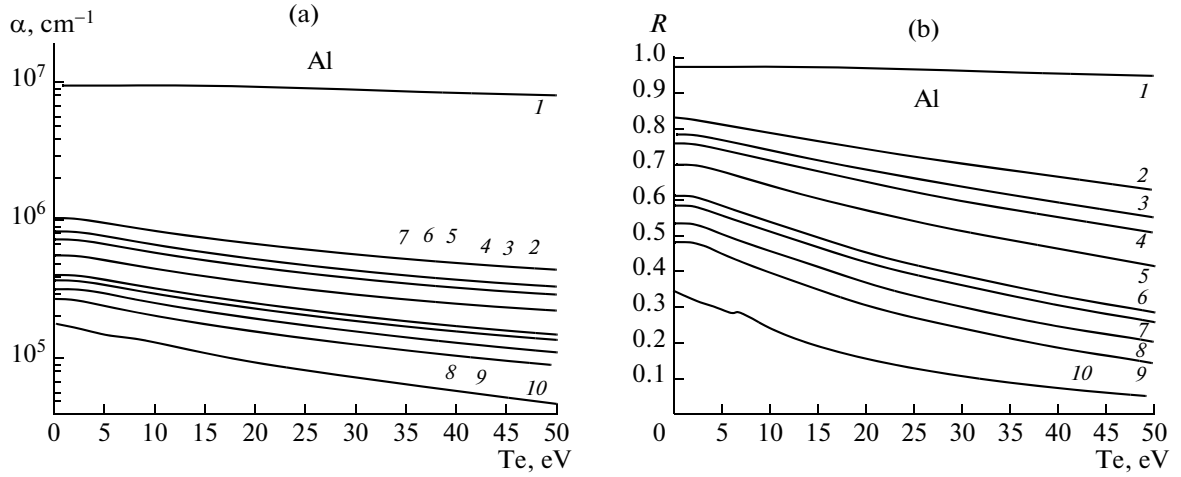

Fig. 3. Temperature dependences of (a) the volume absorption coefficient and (b) the reflexivity capacity of the surface at fixed frequencies: *1.* $\hbar\omega = 0.117$ eV ($\lambda = 10.6$ μm); *2.* $\hbar\omega = 1.17$ eV ($\lambda = 1.06$ μm); *3.* $\hbar\omega = 1.55$ eV ($\lambda = 0.8$ μm); *4.* $\hbar\omega = 1.79$ eV ($\lambda = 0.694$ μm); *5.* $\hbar\omega = 2.43$ eV ($\lambda = 0.51$ μm); *6.* $\hbar\omega = 3.68$ eV ($\lambda = 0.337$ μm); *7.* $\hbar\omega = 4.025$ eV ($\lambda = 0.308$ μm); *8.* $\hbar\omega = 4.999$eV ($\lambda = 0.248$ μm); *9.* $\hbar\omega = 6.42$ eV ($\lambda = 0.193$ μm); *10.* $\hbar\omega = 12.4$ eV ($\lambda = 0.1$ μm).

point $\hbar\omega = \hbar\omega_{Le}$, the real part $\varepsilon_1^l(\omega)$ changes sign, as noted in Fig. 1 by the intersection of two perpendicular dotted lines.

The obtained dependence $\varepsilon_1^l(\omega)$ and $\varepsilon_2^l(\omega)$ were used to determine the dispersion of the real n and imaginary κ parts of refractive index N, with which the frequency dependences of the volume absorption coefficient $\alpha(\omega)$ and reflectance ability of the surface $R(\omega)$ were calculated according to Eqs. (5) and (6) (see Fig. 2). Comparison of the dependences obtained $\alpha(\omega)$ and $R(\omega)$ (solid line) with reference data [3] (dashed lines) (Fig. 2) showed fairly good quantitative agreement in the laser range of frequencies $\hbar\omega \in [0.1{-}10]$ eV. At high frequencies $\hbar\omega > 10$ eV and radiation intensities $G > 10^{13}$ W/cm^2, the contribution of the tunneling and photo effects becomes significant, which is not taken into account in this approach. In these areas, the volume absorption coefficient must be presented as the sum of the coefficients

$$\alpha_\Sigma = \alpha + \alpha_{ph},$$

where α_{ph} coefficient of photoabsorption.

Temperature dependence $\alpha(\omega, T_e)$ and $R(\omega, T_e)$ were calculated for several fixed frequencies corresponding to the radiation wavelengths of the most commonly used lasers. In the laser range $\lambda \in (0.1-10.6)$ μm, ranging from infrared to ultraviolet, both characteristics (Fig. 3) tend to decrease with an increase of the temperature of the electron gas.

CONCLUSIONS

The impact of super (power) ultrashort laser pulses is accompanied by strongly nonequilibrium regions in a solid with hot electrons and a cold lattice. The electronic subsystem can thus be warmed up to temperatures comparable to the Fermi energy or exceeding them. The implementation of computational experiments to study the impact of such regimes requires the determination of the optical, thermal, and other characteristics of the irradiated targets in wide temperature and frequency ranges. In this paper, through the example of calculating the volume absorption coefficient and reflectivity of the aluminum surface, an approach to determine the optical properties of metals at arbitrary temperatures and frequencies is proposed.

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