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DYNAMIC ADAPTATION METHOD FOR RUNNING WAVE PROBLEMS WITH NONLINEAR HEAT CONDUCTION

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Abstract. A class of problems of gas dynamics with nonlinear heat conductivity which solutions are represented as discontinuities or continuous running waves is considered. Numerical solutions are obtained with use of a dynamic adaptation method. The main concept of the method is automatic transformation of the coordinates which is carried out by means of the sought solution. The adaptation function is defined from the quasi-stationarity principle. An arbitrary non-stationary coordinate system allows explicit tracking all discontinuities and continuous running waves. Accuracy of calculations is defined from comparison with the received self-similar solution. Efficiency of the method is defined by the relation of quantity of nodes required for achievement of the same calculation accuracy on grid with dynamic adaptation and on the fixed grid. Dynamic adaptation allows obtaining solution on grids with 30–50 nodes that by 2–3 order is less than the number of nodes on the fixed grids.

1 INTRODUCTION

The problems related to the motion of completely ionized plasma comprise an entire class in gas dynamics. Along with radiation, the basic mechanisms of energy transfer in plasmas are convection and conduction. Accordingly, mathematical models for these problems are underlain by the radiative transfer equations and the fluid dynamic equations with nonlinear heat conduction. Much attention to the influence of nonlinear heat conduction on the interaction between thermal and hydrodynamic processes was given in [1–6]. It was established that problems of this class have widely different solutions, and self-similar solutions were obtained in a relatively narrow range of parameters [1, 4–8]. The interaction between thermal and hydrodynamic fluxes changes qualitatively with varying heat conduction of the medium. Purely hydrodynamic phenomena dominate at low heat conduction. In this case, the heat conduction is a dissipating and smoothing factor. High heat conduction leads to the development of temperature waves [9], which are divided into two different types according to the form of the hydrodynamic motion they cause.

Temperature waves of the first kind (TW-I) are characterized by supersonic heat transfer with temperature waves propagating at a finite velocity against a zero-temperature initial background. An isothermal shock wave can develop behind a TW-I front.

Temperature waves of the second type (TW-II) are characterized by subsonic heat transfer. A TW-II front propagates behind a shock wave and is characterized by a zero heat flux W, a maximum of the density ρ, and a local temperature minimum.

Computationally, the fluid dynamic equations with nonlinear heat conduction represent a complicated problem to solve. A typical solution to such problems has a complex structure and includes strong discontinuities (shock wave fronts), weak discontinuities (thermal wave fronts), and regions of steep temperature, pressure, density, and velocity gradients. The structure of thermal-wave and shock fronts depends on the degree of nonlinearity of the heat equation. Without taking into account the dissipative processes in the medium, a shock wave is a strong discontinuity in all the solution components u, ρ, P, and T. When heat conduction is taken into account, the discontinuity in T(x, t) is eliminated and the temperature front now has an effective width. At low heat conduction, the temperature shock front is nearly quasi-discontinuous. When the thermal conductivity depends strongly (as a power law) on temperature, the effective width of the shock front increases considerably and temperature waves develop whose fronts at the matching point of the solution with the unperturbed front are weak discontinuities.

The presence of discontinuous solutions, steep-gradient regions, and their fast propagation in space impose strict requirements on the efficiency of the computational algorithms used, primarily, on the grid generation principles rather than on the quality of the difference schemes applied.

In this paper, the problem of an accelerating piston is considered in the framework of one-dimensional unsteady fluid dynamics with nonlinear heat conduction (the thermal conductivity λ is a power function of T and ρ: $\lambda(T, \rho) = \lambda_0 T^a \rho^b$. The regime of heat transfer depends on a number of factors. The relation between the thermal and hydrodynamic processes involved is significantly affected by the law of motion of the piston and by the law of heat flux variation on it.

The main goal of this paper is to develop an effective computational algorithm with a controllable grid point distribution and explicitly specified strong and weak discontinuities as applied to fluid dynamics problems with nonlinear heat conduction. In this case, the total number of grid nodes is less by several orders of magnitude than in grids with fixed nodes.

2 MATHEMATICAL STATEMENT OF THE PROBLEM

In the Eulerian formalism, the problem of a piston accelerating in an ideal gas with nonlinear heat conduction is described by the complete system of gas dynamic equations with heat conduction

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) = 0, \tag{1}$$

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}\left(P + \rho u^2\right) = 0, \tag{2}$$

$$\frac{\partial}{\partial t}(\rho\varepsilon)+\frac{\partial}{\partial x}(\varepsilon\rho u)+P\frac{\partial u}{\partial x}+\frac{\partial W}{\partial x}=0, \quad W=-\lambda(\rho,T)\frac{\partial T}{\partial x}, \tag{3}$$

with the equations of state $P=\rho RT$, $\quad \varepsilon=\frac{R}{\gamma-1}T, \quad \gamma=\frac{5}{3}$.

Here, ρ is the density, *u* is the velocity, *P* is the pressure, ε is the internal energy, *T* is the temperature, *R* is the gas constant, γ is the adiabatic index, *W* is heat flux, and λ is the thermal conductivity. It is assumed that λ is a power function of the temperature and density: $\lambda(T,\rho)=\lambda_0 T^a \rho^b$. Parameters a = 5/2, b = 0 are chosen for fully ionized plasma.

Initial conditions. At t = 0, it is assumed that the background temperature, velocity, and pressure are zero and the background density is a constant:

$$u(x,0)=0, \quad P(x,0)=0, \quad T(x,0)=0, \quad \rho(x,0)=\rho_0. \tag{4}$$

Boundary conditions. They are formulated taking into account the fact that strong and weak discontinuities are explicitly tracked in the dynamic adaptation method. The left plane $x=\Gamma_p(t)$ is the surface of the piston. When $W \neq 0$, this is a source of motion and heat. For this reason, two boundary conditions determining the velocity of the piston and the heat flux are specified on the piston surface:

$$u(\Gamma_p(t),t)=v_0 t^n, \; W(\Gamma_p(t),t)=\rho_0 v_0{}^3 t^{3n}. \tag{5}$$

The particular values of a, b, and nare matched with a self-similar solution and will be specified later.

The background values are preserved on the right boundary x= ∞:

$$u(\infty,t)=0, \quad T(\infty,t)=0, \quad \rho(\infty,t)=\rho_0. \tag{6}$$

Since all the perturbations arise on the left boundary (piston surface) $x=\Gamma_p(t)$ and propagate rightward, the unperturbed domain is excluded from consideration in order to reduce the computational costs. For this purpose, the right boundary is shifted toward the left one to be a small distance away from it. When a perturbation arises on the right boundary, it is treated as a free surface ($x=\Gamma_T(t)$) propagating at the velocity of thermal or gas dynamic perturbations. In problems with nonlinear heat conduction and a zero temperature background, the new boundary $x=\Gamma_T(t)$ always coincides with a temperature wave front, which is a weak discontinuity whose propagation velocity v_T is determined by a relation derived from the equation of motion in the moving frame of reference. The other conditions are transferred from (6) without change:

$$x=\Gamma_T(t): \qquad v_T=\frac{1}{\rho_0}\frac{\partial P}{\partial u}, \quad u(\Gamma_T(t),t)=0, \quad T(\Gamma_T(t),t)=0, \quad \rho(\Gamma_T(t),t)=\rho_0. \tag{7}$$

Relations on the shock front. Since the temperature across the front shock $x=\Gamma_W(t)$ is continuous, we write three conservation laws (Rankine–Hugoniot relations):

$$
\begin{gathered}
\rho_-(u_- - v_W) = \rho_+(u_+ - v_W) = D_M \\
P_- + \rho_-(u_- - v_W)^2 = P_+ + \rho_+(u_+ - v_W)^2 \\
\varepsilon_- + P_-/\rho_- + \frac{(u_- - v_W)^2}{2} + \frac{W_-}{D_M} = \varepsilon_+ + P_+/\rho_+ + \frac{(u_+ - v_W)^2}{2} + \frac{W_+}{D_M}.
\end{gathered} \tag{8}
$$

Here, the minus and plus indices denote the variables on different sides of the shock wave, v_W is the velocity of the shock wave, and D_M is the mass flux across the shock front.

3 ARBITRARY NONSTATIONARY SYSTEM OF COORDINATES

According to the dynamic adaptation method [10-13], we proceed to an arbitrary non-stationary coordinate system. In the new variables (q, τ), system (1)–(3) becomes

$$\frac{\partial}{\partial\tau}(\psi\rho)+\frac{\partial}{\partial q}(\rho(u+Q))=0, \tag{9}$$

$$\frac{\partial}{\partial\tau}(\psi\rho u)+\frac{\partial}{\partial q}(P+\rho u(u+Q))=0, \tag{10}$$

$$\frac{\partial}{\partial\tau}(\psi\rho\varepsilon)+\frac{\partial}{\partial q}(\varepsilon\rho(u+Q))+P\frac{\partial u}{\partial q}+\frac{\partial W}{\partial q}=0, \quad W=-\frac{\lambda(\rho,T)}{\psi}\frac{\partial T}{\partial q}, \tag{11}$$

$$\frac{\partial\psi}{\partial\tau}=-\frac{\partial Q}{\partial q}, \tag{12}$$

On proceeding to an arbitrary non-stationary coordinate system, the original Euler equations (1)–(3) are transformed into extended model (9)–(12), which has been supplemented by the inverse transformation equation (12). Initial and boundary conditions (4)–(7) are

$$u(q,0)=0, \quad P(q,0)=0, \quad T(q,0)=0, \quad \rho(q,0)=\rho_0, \quad \psi(q,0)=1 \quad at\ \tau=0, \tag{13}$$

$$u(\Gamma_p,\tau)=v_0\tau^n, \quad W(\Gamma_p,\tau)=\rho_0 v_0^{\,3}\tau^{3n}, \quad Q(\Gamma_p,\tau)=-v_0\tau^n, \quad at\ q=\Gamma_p \tag{14}$$

$$Q(\Gamma_T,\tau)=-\frac{1}{\rho_0}\cdot\frac{\partial P}{\partial u}, \ u(\Gamma_T,\tau)=0, \ T(\Gamma_T,\tau)=0, \ \rho(\Gamma_T,\tau)=\rho_0 \quad at\ q=\Gamma_T. \tag{15}$$

Here $q=\Gamma_p$ is the left plane is the surface of the piston. The new boundary $q=\Gamma_T$ always coincides with a temperature wave front, which is a weak discontinuity whose propagation velocity $v_T=-Q(\Gamma_T,\tau)$ is determined by a relation derived from the equation (10). In the nonstationary coordinate system, discontinuities are explicitly introduced in the solution and, after a shock wave appears, system (9)–(12) is solved in two subdomains divided by the shock front. At the front $q=\Gamma_W$, the resulting solutions are matched using the Rankine–Hugoniot conditions (8):

$$\left.\begin{aligned} \rho_-(u_-+Q_W)=\rho_+(u_++Q_W)=D_M \\ P_-+\rho_-(u_-+Q_W)^2=P_++\rho_+(u_++Q_W)^2 \\ W_-+0.5\rho_-(u_-+Q_W)^3=W_++0.5\rho_+(u_++Q_W)^3 \\ Q_W=-v_W \end{aligned}\right\} \text{ at } q=\Gamma_W. \quad (16)$$

4 CHOICE OF THE ADAPTATION FUNCTION

The grid point distribution in the dynamic adaptation method is controlled using the adaptation function *Q*. In the case of steep-gradient solutions, this function is usually determined from the quasi-stationarity principle [12-14], according to which we choose a non-stationary coordinate system in which all the physical processes proceed as steady-state ones and the corresponding time derivatives are relatively small. Setting the time derivatives in the equations equal to zero yields the sought adaptation function.

The general solution to the complete system of fluid dynamics equations (9)–(16) is determined by the sum of the velocity, density, and temperature. These functions have different (frequently oppositely directed) spatiotemporal distributions. A controllable grid point distribution for the system of equations must take into account the features of the spatiotemporal distributions for all the solution components.

In the general case, the adaptation function in fluid dynamics problems can be determined using the entire system of equations [10, 13]. In this paper, the function *Q* is found from energy equation (11), whose solution depends on the velocity, density, and heat conduction. In non-conservative form, the energy equation is

$$\frac{\partial\varepsilon}{\partial\tau}+\frac{(u+Q)}{\psi}\frac{\partial\varepsilon}{\partial q}+\frac{P}{\rho\psi}\frac{\partial u}{\partial q}+\frac{\partial W}{\partial q}=0.$$

Based on the quasi-stationarity principle, we set $\partial\varepsilon/\partial\tau=0$ to obtain the equation

$$\rho(u+Q)\frac{\partial\varepsilon}{\partial q}+P\frac{\partial u}{\partial q}+\frac{\partial W}{\partial q}=0 \quad (17)$$

By taking into account the particular form of the equations of state $P=\rho RT$, $\varepsilon=\frac{R}{\gamma-1}T$ and differentiating the heat flux $W=-\frac{\lambda(\rho,T)}{\psi}\cdot\frac{\partial T}{\partial q}$, the function *Q* is determined by simple rearrangements in (17)

$$Q=-\left[u+C_1T\frac{\partial u}{\partial q}\Big/\left(\frac{\partial T}{\partial q}+re\right)\right]+$$
$$+\left[\frac{C_2}{\rho\psi}\left(\frac{\partial\lambda}{\partial\rho}\frac{\partial\rho}{\partial q}+\frac{\partial\lambda}{\partial T}\frac{\partial T}{\partial q}+\lambda(\rho,T)\frac{\partial^2T}{\partial q^2}\Big/\left(\frac{\partial T}{\partial q}+re\right)\right)\right]+ \tag{18}$$
$$+\left[C_2\frac{\lambda(\rho,T)}{\rho}\frac{\partial}{\partial q}\left(\frac{1}{\psi}\right)\right],\qquad C_1=(\gamma-1),\quad C_2=\frac{(\gamma-1)}{R},$$

where *re* is a regularizing constant that is a lower bound for the derivative as it tends to zero.

After the difference approximation, the first square bracket in (18) exerts a contraction effect on the grid points in u and *T*. The second square bracket takes into account the influence of nonlinear heat conduction and exerts a contraction effect with respect to ρ and *T*. The last term is of the diffusion type. If $\lambda(\rho, T) \neq 0$, it has a smoothing effect and, in particular, prevents the intersection of grid point trajectories.

The features of the class of problems under consideration are determined by two factors. The first is that the thermal conductivity is a power function of the temperature. At low temperatures (near zero), since the thermal conductivity is low, the dissipating effect of the diffusion term decreases sharply and may become insufficient for an optimal grid point distribution. The second factor is that the original problem is represented in the form of a free-surface problem. The original domain may then increase by many orders of magnitude. Accordingly, the values of ψ increase as well, which also strongly reduces the diffusion component. To eliminate these effects, it is reasonable to supplement Q with a function obtained from the diffusion approximation [10] taking into account the presence of moving boundaries:

$$Q=-D\frac{\partial\psi}{\partial q},$$

where *D* is the diffusivity. Its value is determined by the geometric size of a cell (the mesh size *h*), by the velocity of the boundary points (υ_l, υ_r), and by the minimum of the function ($\psi_{\min}$) over the entire domain:

$$D=\frac{h\left|\max(\upsilon_l,\upsilon_r)\right|}{\psi_{\min}}.$$

Additionally, it is reasonable to represent the ratio of two temperature derivatives in Eq. (18) in the form of the derivative of a slowly varying logarithmic function:

$$\frac{\partial^2T}{\partial q^2}\Big/\left(\frac{\partial T}{\partial q}+re\right)=\frac{\partial}{\partial q}\left(\ln\left(\left|\frac{\partial T}{\partial q}\right|+re\right)\right)$$

In view of the features described above, the adaptation function can be finally written as

$$Q=-\left[u+C_1T\frac{\partial u}{\partial T}\right]+$$

$$+\left[\frac{C_2}{\rho\psi}\left(\frac{\partial\lambda}{\partial\rho}\frac{\partial\rho}{\partial q}+\frac{\partial\lambda}{\partial T}\frac{\partial T}{\partial q}+\lambda(\rho,T)\frac{\partial}{\partial q}\left(\ln\left(\left|\frac{\partial T}{\partial q}\right|+re\right)\right)\right)\right]-\left[\left(\frac{C_2\lambda(\rho,T)}{\rho\psi^2}+D\right)\frac{\partial\psi}{\partial q}\right] \quad (19)$$

$$C_1=(\gamma-1),\quad C_2=\frac{(\gamma-1)}{R}$$

5 SELF-SIMILAR SOLUTION

To estimate the advantages of the dynamic adaptation method, the resulting numerical solution should be compared with the analytical one (if any) or with a solution obtained in another way. A widespread approach is based on a comparison with self-similar solutions. Fluid dynamics problems with nonlinear heat conduction have been well studied and the self-similar solutions available for them comprise a relatively large class [4-8].

Following the technique described in [8], we find a self-similar solution to problem (1)–(3) with a nonlinear thermal conductivity represented as a power function of the temperature and density,

$$\lambda=\lambda_0T^a\rho^b,\qquad a>0,\, b\le 0, \quad (20)$$

and with boundary conditions (5) with a nonlinear heating flux on the boundary $x=\Gamma_p(t)$:

$$u(\Gamma_p(t),t)=v_0t^n,\ W(\Gamma_p(t),t)=\rho_0 {v_0}^3t^{3n}.$$

To determine a self-similar solution, we need to find *a*, *b*, and *n* for which the given solution exists, to set up a system of ODEs in similarity variables, and to solve it numerically.

Gas dynamic equations (1)–(3) with heat conduction can be analyzed as follows. The physical dimensions of the functions and constants used in the problem are

$$\begin{cases}[\rho]=M\,L^{-3}\\ [u]=L\tilde{T}^{-1}\\ [T]=\tilde{C}\\ [W]=M\,\tilde{T}^{-3}\\ [\lambda]=M\,L\tilde{T}^{-3}\,\tilde{C}^{-1}\end{cases}\quad\Rightarrow\quad\begin{cases}[R]=L^2\,\tilde{T}^{-2}\,\tilde{C}^{-1}\\ [\rho_0]=M\,l^{-3}\\ [v_0]=L\tilde{T}^{-1-n}\\ [\lambda_0]=M^{1-b}\,L^{1+3\cdot b}\,\tilde{T}^{-3}\,\tilde{C}^{-1-a}\end{cases} \quad (21)$$

where $M, L, \tilde{T}, \tilde{C}$ are the dimensions of mass, length, time, and temperature.

It is well known [15] that a solution is self-similar if all the governing dimensional parameters of the problem (in our case, $R, \rho_0, v_0, \lambda_0$) include *k* – 1 constants with independent dimensions, where *k* is the number of basic dimensional units. Since *k* = 4 in the problem under study, one of the four governing parameters must be linearly dependent on the other three. Choosing v_0, ρ_0, R to be constant parameters with independent dimensions, we determine a, b, and n for which λ_0 is expressed in terms of these parameters.

Let $\lambda_0 = \hat{\lambda}_0 \cdot {v_0}^x \cdot {\rho_0}^y \cdot R^z$, where $\hat{\lambda}_0$ is a dimensionless constant. Then, taking into account (21), the system of correspondence of each dimension to λ_0 is written as

$$\begin{pmatrix} M \\ L \\ \tilde{T} \\ \tilde{C} \end{pmatrix} \Leftrightarrow \begin{cases} y = 1 - b \\ x - 3y + 2z = 1 + 3b \\ (1+n)x + 2z = 3 \\ z = 1 + a \end{cases} \Rightarrow \begin{cases} y = 1 - b \\ x + 2z = 4 \\ nx = -1 \\ z = 1 + a \end{cases} \Rightarrow 2a = 2 + n^{-1}.$$

Setting a = 5/2 and b = 0, we see that the self-similarity condition is n = 1/3.

To derive a system of ODEs, we introduce the dimensionless functions f, α, δ, w,

$$f(s) = \frac{RT(x,t)}{{v_0}^2 t^{2\cdot n}}, \quad \alpha(s) = \frac{u(x,t)}{v_0 t^n}, \quad \delta(s) = \frac{\rho(x,t)}{\rho_0}, \quad w(s) = \frac{W(x,t)}{\rho_0 {v_0}^3 t^{3\cdot n}},$$

where the formulas for the transition to the coordinate s are given by

$$\frac{\partial s}{\partial x} = \frac{\delta}{v_0 t^{n+1}} \text{ и } \frac{\partial s}{\partial t} = \frac{ds}{dt} - u\frac{\partial s}{\partial x} = -(n+1)s t^{-1} - \alpha \delta t^{-1}.$$

Then, for example, for the equation in (1), we have

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) = 0 \Rightarrow \frac{\partial \rho}{\partial s} \cdot \frac{\partial s}{\partial t} + \frac{\partial}{\partial s}(\rho u)\frac{\partial s}{\partial x} = 0 \Rightarrow$$

$$\rho_0 \delta' \left(-(n+1)s t^{-1} - \alpha \delta t^{-1}\right) + \left(\rho_0 \delta' \alpha v_0 t^n + \rho_0 \delta \alpha' v_0 t^n\right) \frac{\delta}{v_0 t^{n+1}} = 0 \Rightarrow$$

$$-(n+1)s\delta' + \delta^2 \alpha' = 0$$

Proceeding in a similar manner with the other equations in (2)–(3) and with the heat equation involving the thermal conductivity given by (20), we obtain

$$(n+1)s\delta' = \delta^2 \alpha'$$

$$n\alpha - (n+1)s\alpha' = -(f\delta)'$$

$$\frac{1}{\gamma - 1}\left(2nf - (n+1)s f'\right) = -f\delta\alpha' - w'$$

$$w = -\hat{\lambda}_0 f^a \delta^{b+1} f'$$

or, in view of a = 5/2, b = 0, n = 1/3, and γ = 5/3,

$$\begin{aligned} &\frac{4s\delta'}{3} = \delta^2 \alpha' \\ &\frac{\alpha}{3} - \frac{4s\alpha'}{3} = -(f\,\delta)' \\ &f - 2s\,f' = -f\,\delta\alpha' - w' \\ &w = -\hat{\lambda}_0\, f^{5/2}\,\delta\, f' \end{aligned} \tag{22}$$

The boundary conditions are

$s = 0 \qquad \alpha = 1,\ w = 1$

$s = \infty \qquad \alpha = 0,\ w = 0,\ f = 0,\ \delta = 1.$

By introducing the auxiliary functions

$$F = -\frac{w}{\hat{\lambda}_0\, f^{5/2}\,\delta},\ \Delta = \frac{16}{9}s^2 - \delta^2 f \ \text{ and } \ \varphi = \frac{1}{3}\alpha + \delta F$$

system (22) can be rewritten in a somewhat different form:

$$\begin{aligned} &\alpha' = \frac{4}{3}\frac{s\varphi}{\Delta} \\ &\delta' = \frac{\varphi\,\delta^2}{\Delta} \\ &w' = 2sF - f - f\,\delta\frac{4}{3}\frac{s\varphi}{\Delta} \\ &f' = F \end{aligned} \tag{23}$$

Before solving this system, we make the following two remarks. First, there exists a point s_0 (thermal wave front) at which the function f' is discontinuous. To the right of this point, all the functions have initially given values. Second, the expression for Δ implies that $\Delta < 0$ at $s = 0$ and $\Delta > 0$ at $s = s_0$. Moreover, there is no point q_1 at which $\Delta = 0$ [8]. Therefore, Δ and all the other unknown functions, except for f, are discontinuous at this point. The Rankine–Hugoniot relations at this discontinuity can be written in similarity variables:

$$\begin{aligned} &\frac{\delta_1}{\delta_2} = \frac{36\, f_1\,\delta_1^{\,2}}{49\, s_1^{\,2}} = \Theta \\ &\alpha_2 = \alpha_1 + \frac{7s_1}{6\delta_1}(1-\Theta) \\ &f_2 = f_1 \\ &w_2 = w_1 - 0.5\,\delta_1\left(\frac{7s_1}{6\delta_1}\right)^3\left(1-\Theta^2\right) \end{aligned} \tag{24}$$

The values of s_0 and s_1 are determined by numerically solving system (23) with the corresponding boundary conditions specified at $s = 0$ and $s = s_0$.

The algorithm for solving system (23) can be described as follows. For a chosen value of s_0, we find s_1 such that the dimensionless function α obtained by solving (23), (24) with $s = 0$ becomes equal to the value specified in the boundary conditions ($\alpha = 1$). If $w > 1$, then we choose a new value of s_0 that is smaller than the previous one. If $w < 1$, the new value of s_0 used is greater than the previous one. The iteration continues until $w = 1$ to within the prescribed accuracy, in which case the solution is regarded as found.

We found two self-similar profiles for various thermal conductivities: $\lambda_0 = \{1, 10\}$ (R = 1).

6 SIMULATION RESULTS

The problem of a piston accelerating in a medium with nonlinear heat conduction was simulated by solving system (9)–(12) with initial conditions (13), boundary conditions (14)–(15), Rankine–Hugoniot relations (16), and adaptation function (19). Since the solution was compared with self-similar profiles, the problem was solved in nondimensionalized variables with the constants corresponding to the self-similar solution: $R = 1$, $\rho_o = 1$, $v_o = 1$, $n = 1/3$, $a = 5/2$, $b = 0$.

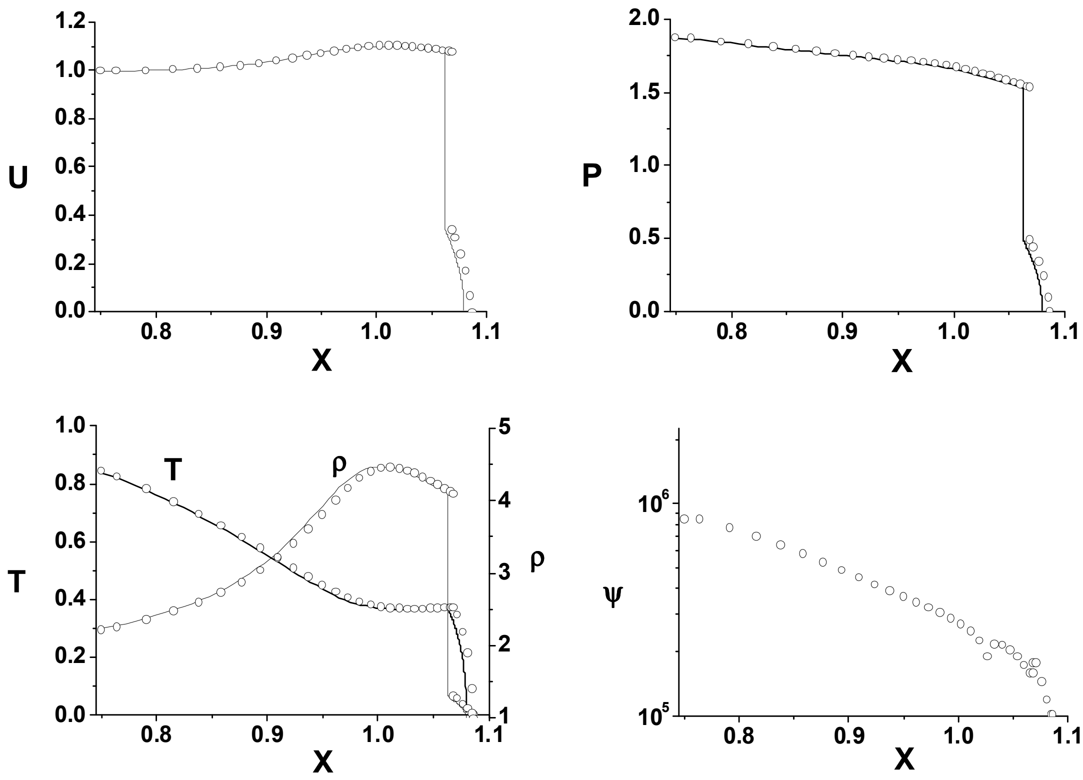

Fig. 1

TW-II is formed if λ_0 is smaller than some dimensionless constant $\lambda_0 < \lambda^*$ (i.e., $\lambda_0 \geq \lambda^*$, where $\lambda^* \approx 3$ in the case under study), and a TW-I is formed if $\lambda_0 \geq \lambda^*$. We considered a version of TW-II with $\lambda_0 = 1$ (Figs. 1) and version of TW-I with $\lambda_0 = 10$ (Figs. 2).

The TW-II with $\lambda_0 = 1$ is characterized by subsonic heat transfer. For its simulation, we used a grid with the total number of nodes $N = 30$, of which 25 were located between the piston and the shock wave and the other 10 between the shock wave and the outer boundary. Figure 1 show the spatial profiles of the gas dynamic functions and the temperature at the times $t = 1$.

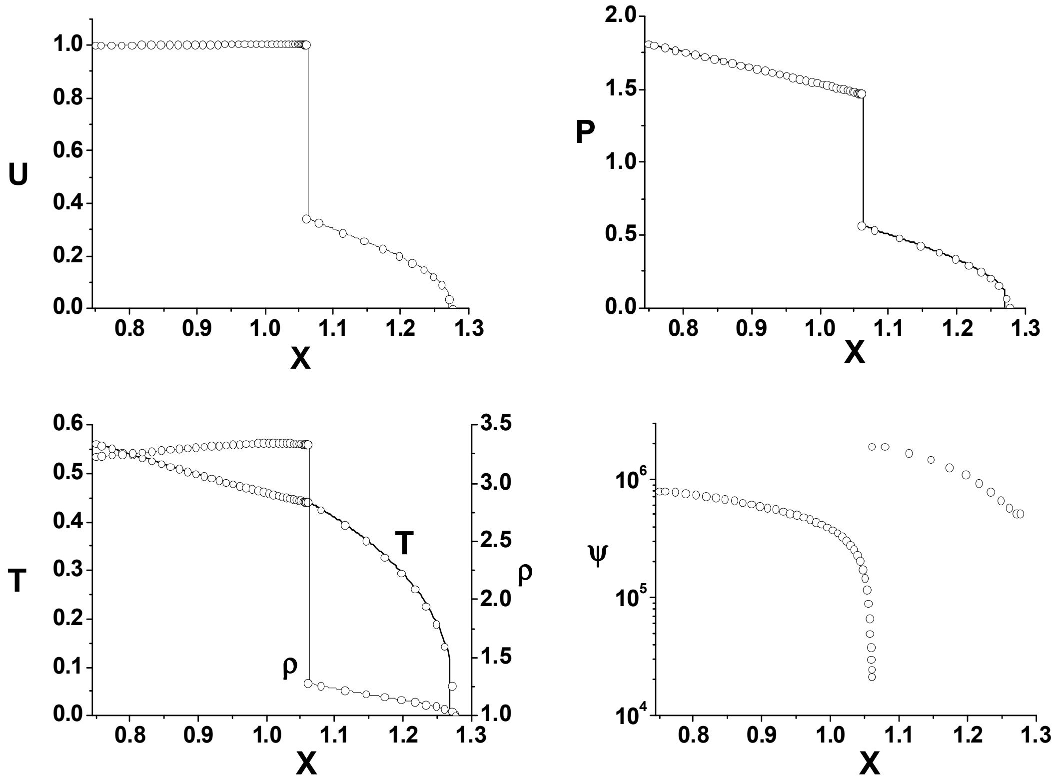

Fig. 2

Figure 2 displays the spatial profiles of the gas dynamic functions, temperature, and ψ for TW-I with $\lambda_0 = 10$ TW-I after the numerical solution has reached the self-similar one (at $t = 1$).

7 CONCLUSIONS

The dynamic adaptation method was applied to gas dynamic simulations with nonlinear heat conduction. The major features of the solution to the fluid dynamic equations with nonlinear heat conduction include the presence of three moving boundaries and two (TW-I regime) or three (TW-II regime) regions of rapid variations in all the solution functions.

For this class of problems, an adaptation function was proposed that controls the grid point distribution depending on the features of the solution. The adaptation function has a complex structure and consists of several terms. Some of them are determined by the diffusion approximation and take into account the variations in the size of the computational domain caused by the motion of the piston and by propagation of weak and strong discontinuities. The other terms are determined by the quasi-stationarity principle and are responsible for mesh refinement in regions with steep gradients of temperature, density, and velocity.

In the course of solving the problem, the numerical solution approached the self-similar profile and ultimately coincided with it. This suggests that the numerical results are of high quality and that the dynamic adaptation method can be applied to gas dynamic simulations with nonlinear heat conduction.

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